May 31, 2014

Example of 2D Steady Seepage Flow Analysis
(Seepage flow analysis of Dams)

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- This document was created by TEX.
- Regarding the figure such as Mesh, contour map and velocity vector map, 'eps' files made by 'GMT' were read by TEX.
- Regarding graphs, 'eps' files made by 'gnuplot' were read by TEX.

1. Homogeneous embankment dam

The seepage flow discharge by seepage flow FEM was compared with it by Casagrande's method.

1.1 Casagrande's method

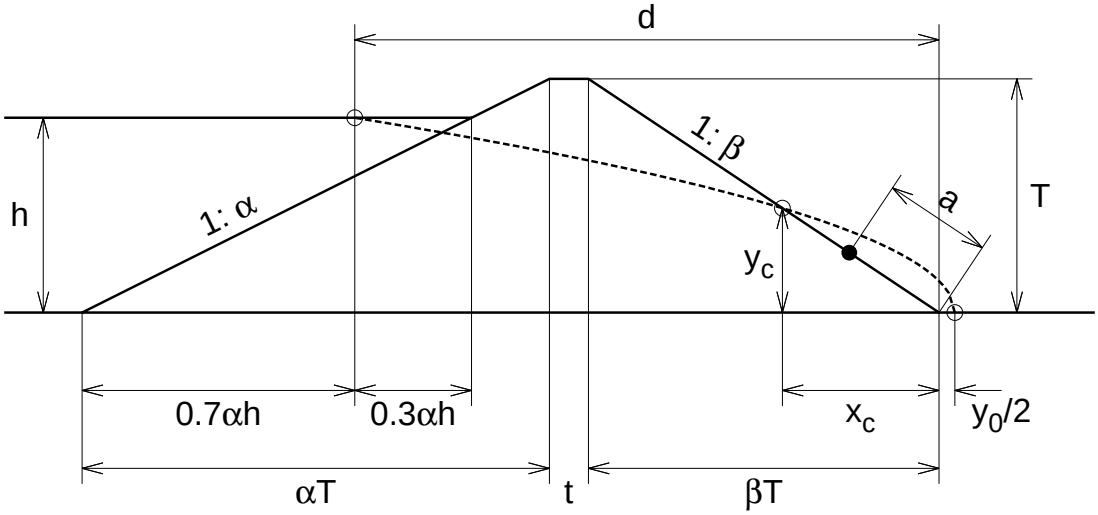


Fig.1 Casagrande's model

T : Dam height
t : Width of dam top
α : Gradient of upstream slope $(1 : \alpha)$
β : Gradient of downstream slope $(1 : \beta)$
h : Water depth of upstream
k : Permeability coefficient of dam body

a. First approximation

$$d = T(\alpha + \beta) + t - 0.7 \cdot \alpha \cdot h$$
$$a = \sqrt{d^2 + h^2} - \sqrt{d^2 - h^2 \cdot \beta^2}$$
$$q = k \cdot \frac{a}{1 + \beta^2}$$

b. Second approximation

$$d = T(\alpha + \beta) + t - 0.7 \cdot \alpha \cdot h$$
$$y_0 = \sqrt{d^2 + h^2} - d$$
$$x_c = \frac{\beta^2 h^2 + \sqrt{\beta^2 h^2 \{\beta^2 h^2 + 2(d + y_0/2) y_0\}}}{2(d + y_0/2)}$$
$$y_c = \frac{x_c}{\beta}$$
$$S = \sqrt{(d - x_c)^2 + (h - y_c)^2} + \sqrt{{x_c}^2 + {y_c}^2}$$
$$a = S - \sqrt{S^2 - h^2(1 + \beta^2)}$$
$$q = k \cdot \frac{a}{1 + \beta^2}$$

Since first approximation gives the result of safety side based on trial calculation, first approximation was used on the calculations in this document.

1.2 Result of seepage flow FEM

Conditions for analysis and outline of results are shown below.

Table1 Conditions for analysis

Dam shape				Water level		Material properties		
Height (m)	Top width (m)	Gradient of slope	Bottom width (m)	Upstream (m)	Downstream (m)	Permeability k (m/s)	α (m^{-1})	m
20.0	2.0	1:0.1	6.0	18.0	0.0	1×10^{-5}	0.1	0.7
20.0	2.0	1:0.5	22.0	18.0	0.0	1×10^{-5}	0.1	0.7
20.0	2.0	1:1.0	42.0	18.0	0.0	1×10^{-5}	0.1	0.7
20.0	2.0	1:1.5	62.0	18.0	0.0	1×10^{-5}	0.1	0.7
20.0	2.0	1:2.0	82.0	18.0	0.0	1×10^{-5}	0.1	0.7
20.0	2.0	1:2.5	102.0	18.0	0.0	1×10^{-5}	0.1	0.7
20.0	2.0	1:3.0	122.0	18.0	0.0	1×10^{-5}	0.1	0.7

Table2 Result of FEA

3 node element

Slope	Angle (°)	Bottom width (m)	Seepage point from USE (m)	Seepage point from DSE (m)	Distance a (m)	Discharge (m^3/s/m)
1:0.1	84.3	6.0	4.3	1.7	17.1	0.3542×10^{-3}
1:0.5	63.4	22.0	15.0	7.0	15.6	0.1178×10^{-3}
1:1.0	45.0	42.0	31.0	11.0	15.6	0.7112×10^{-4}
1:1.5	33.7	62.0	46.0	16.0	19.2	0.5327×10^{-4}
1:2.0	26.6	82.0	60.5	21.5	24.0	0.4256×10^{-4}
1:2.5	21.8	102.0	75.0	27.0	29.1	0.3550×10^{-4}
1:3.0	18.4	122.0	90.0	32.0	33.7	0.3093×10^{-4}

4 node element

Slope	Angle (°)	Bottom width (m)	Seepage point from USE (m)	Seepage point from DSE (m)	Distance a (m)	Discharge (m^3/s/m)
1:0.1	84.3	6.0	4.3	1.7	17.1	0.3533×10^{-3}
1:0.5	63.4	22.0	15.2	6.8	15.2	0.1169×10^{-3}
1:1.0	45.0	42.0	30.5	11.5	16.3	0.6957×10^{-4}
1:1.5	33.7	62.0	46.0	16.0	19.2	0.5084×10^{-4}
1:2.0	26.6	82.0	61.0	21.0	23.5	0.4030×10^{-4}
1:2.5	21.8	102.0	76.0	26.0	28.0	0.3345×10^{-4}
1:3.0	18.4	122.0	93.0	29.0	30.6	0.2864×10^{-4}

Output samples are shown in **Fig4∼7**, **Fig8∼9**. The distance a from the edge of downstream of dam to seepage point was obtained by visual checking of the figures such as **Fig8** for each case.

1.3 Result of comparison

Following matters are cleared from figures.

- ◯ The seepage flow discharge Q by FEM is greater than it by Casagrande's method for each case.
- ◯ If the slope becomes steep, the difference of discharge between FEM and Casagrande's method becomes large.
- ◯ The distance a from downstream edge of dam to seepage point by FEM is greater than it of Casagrande's method for each.
- ◯ If the slope becomes gentle, the difference of distance 'a' between FEM and Casagrande's method becomes large.

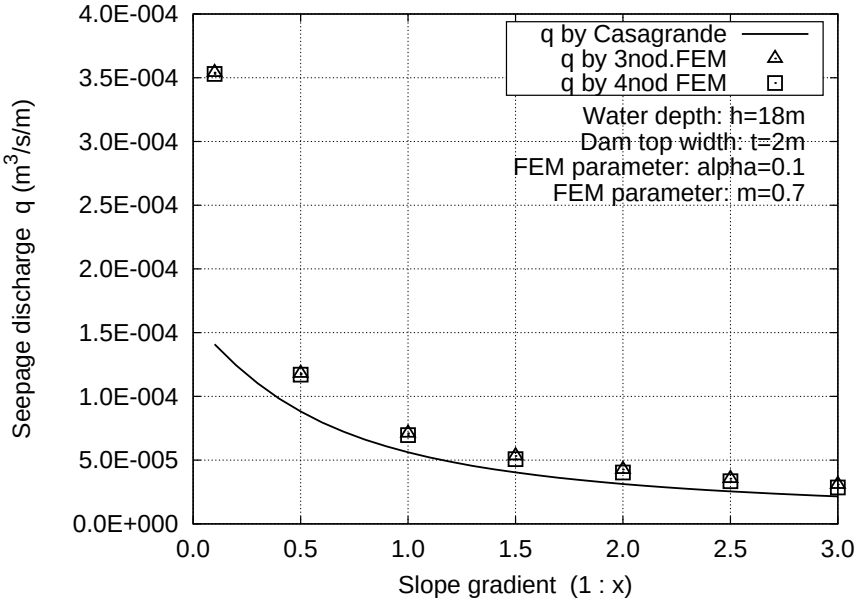


Fig.2 Discharge Q-Slope relationship on Homogeneous embankment dam

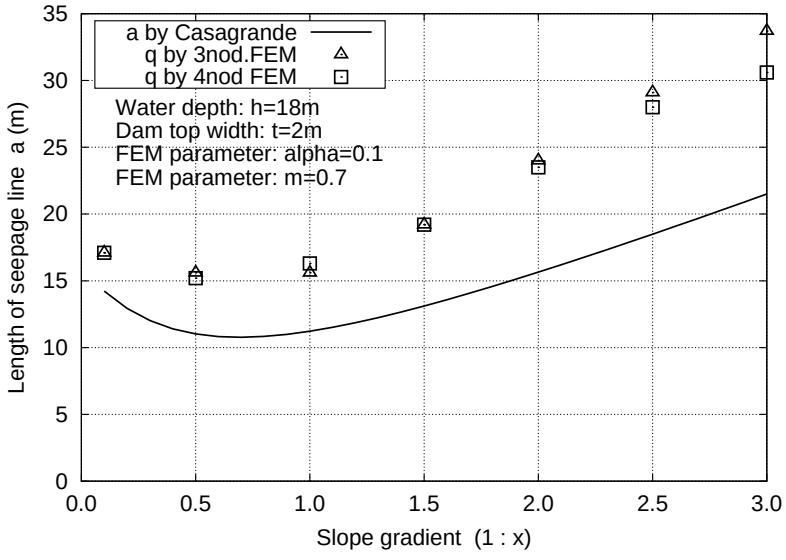
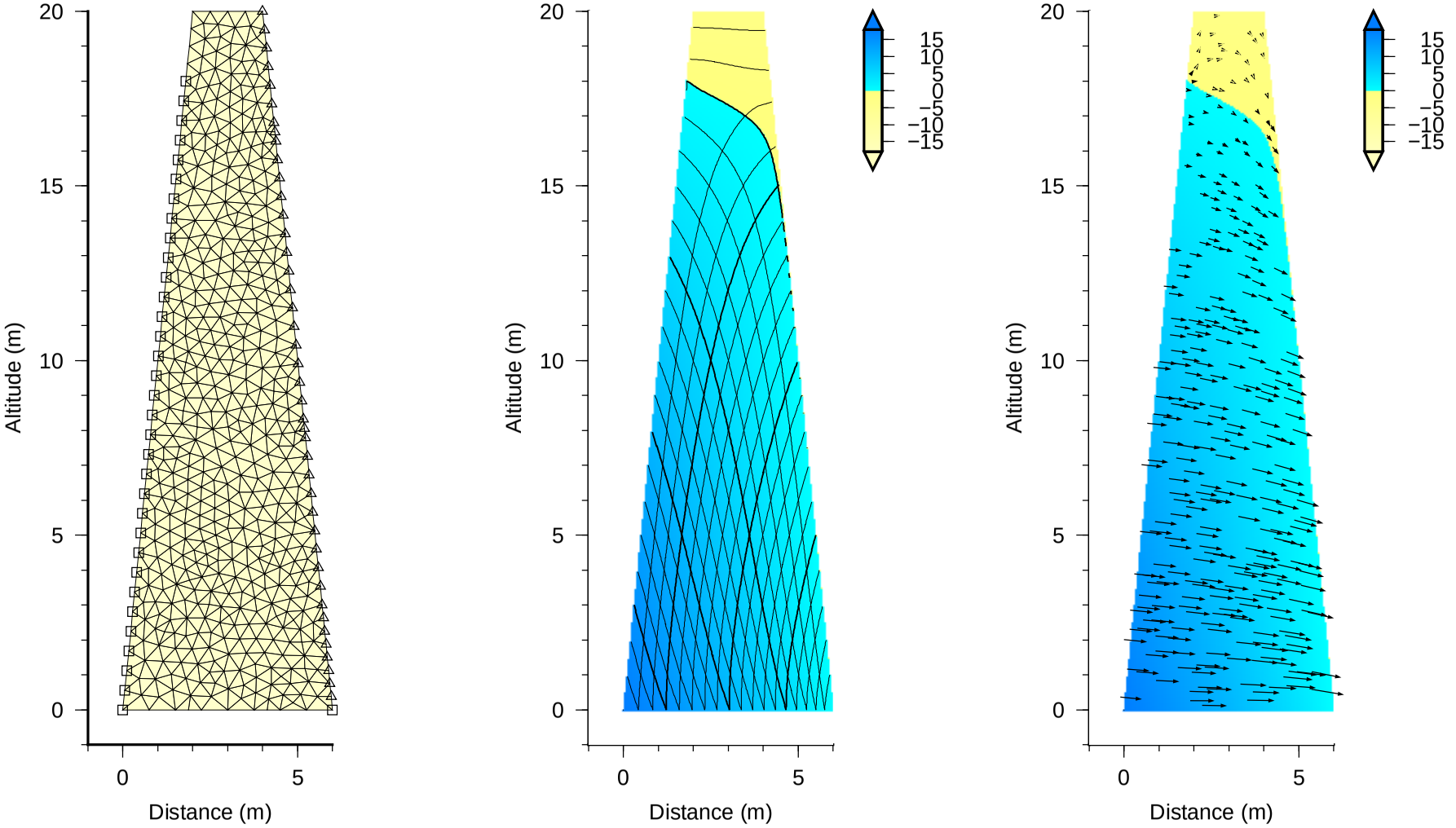
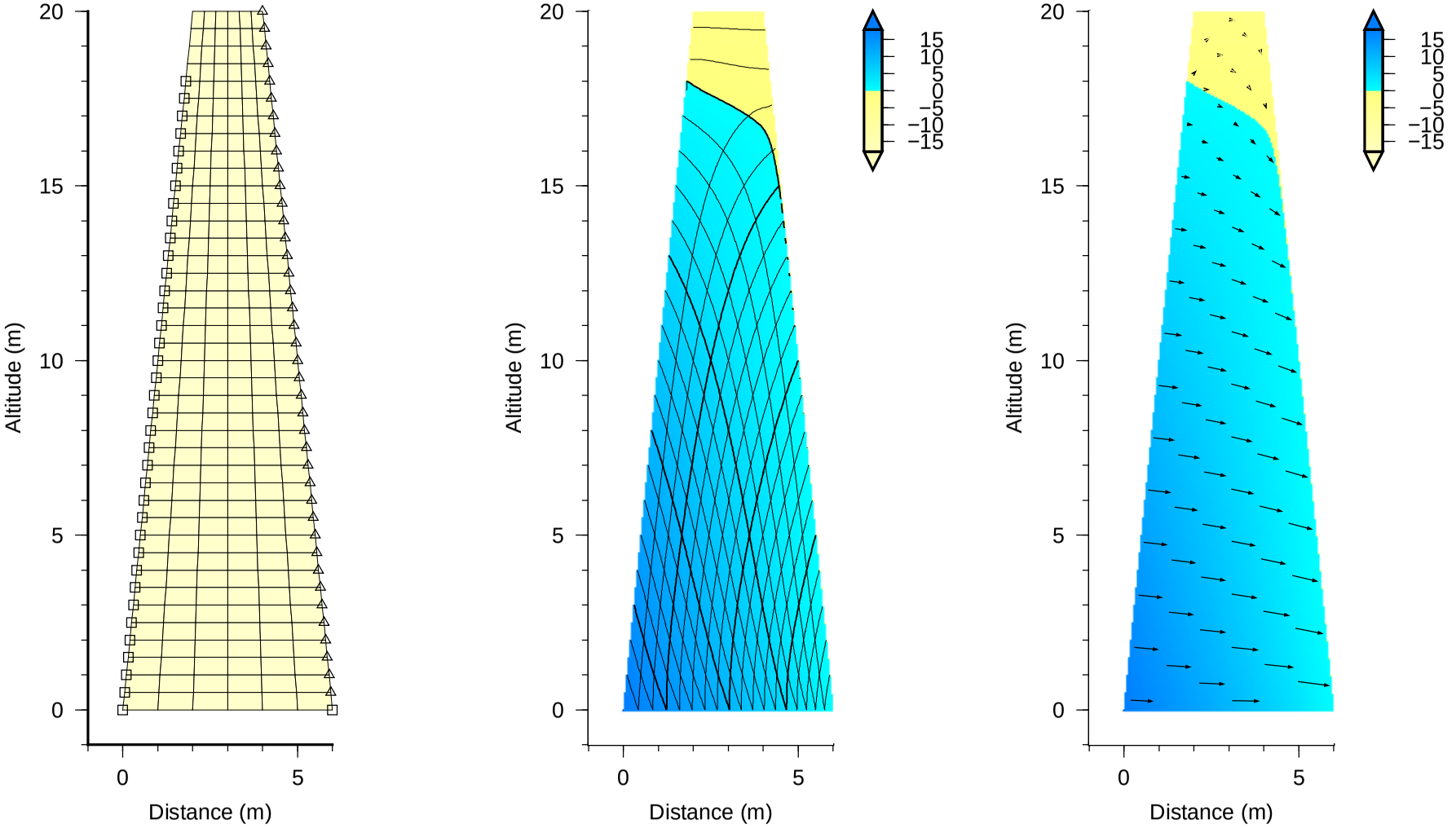


Fig.3 Distance a-Slope relationship on Homogeneous embankment dam



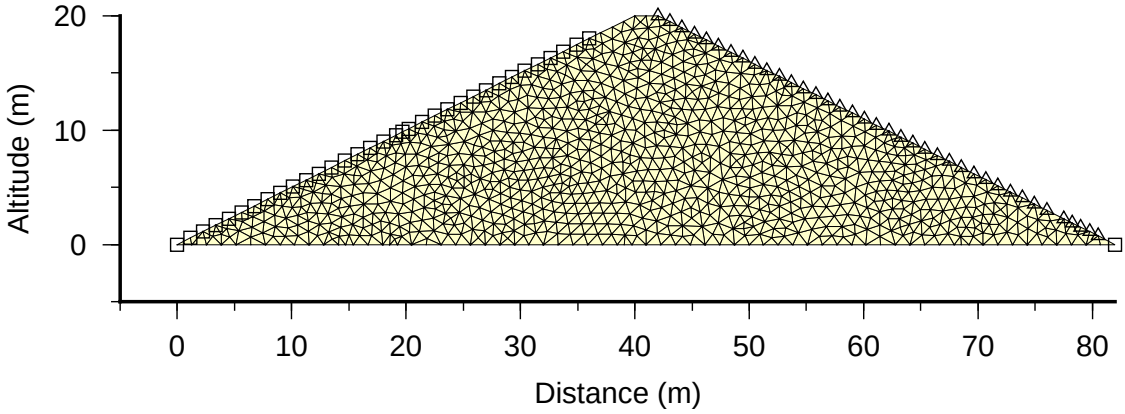
Mesh **Pressure head contour map** **velocity vector map**

Fig.4 Output sample by 3 node element (slope 1:0.5)

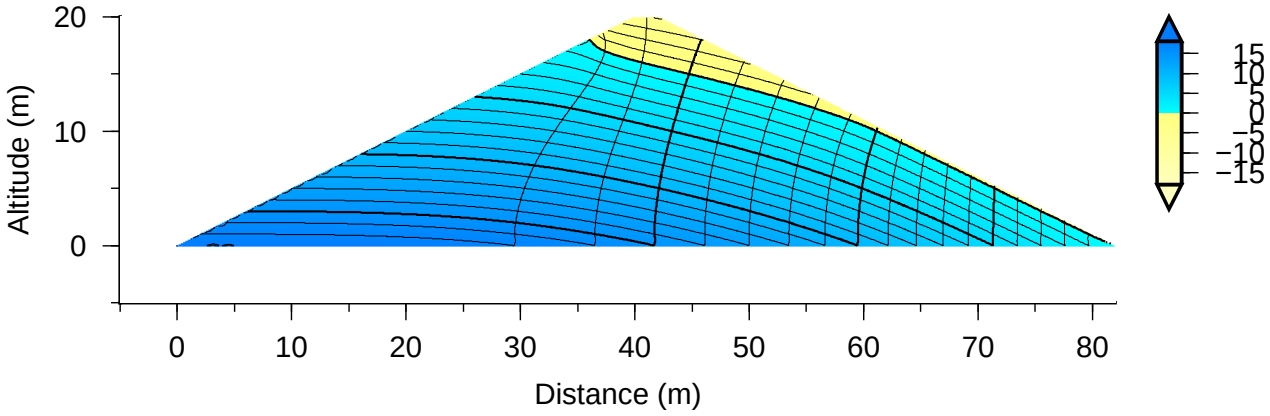


Mesh **Pressure head contour map** **velocity vector map**

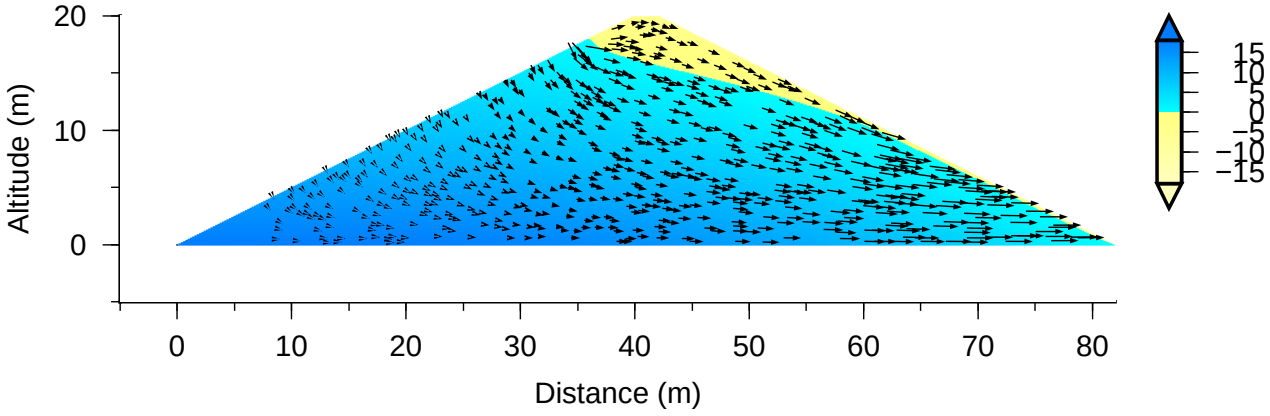
Fig.5 Output sample by 4 node element (slope 1:0.5)



Mesh

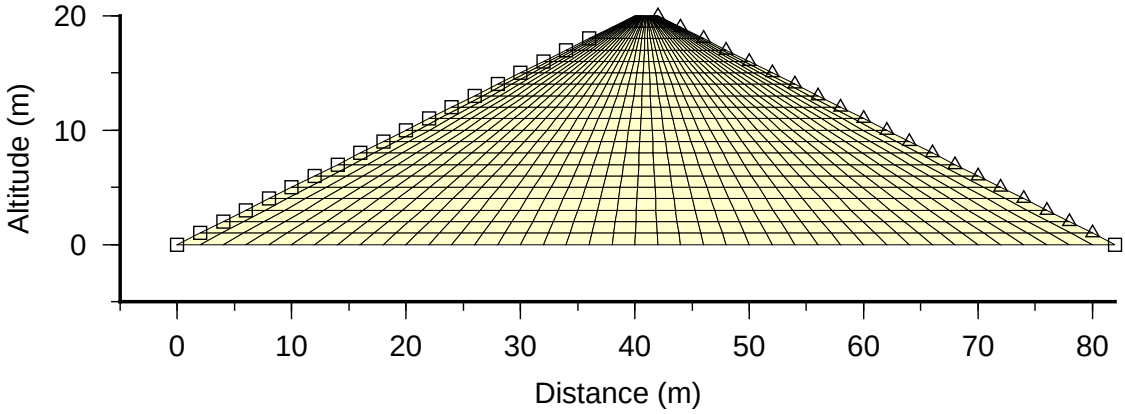


Pressure head contour map

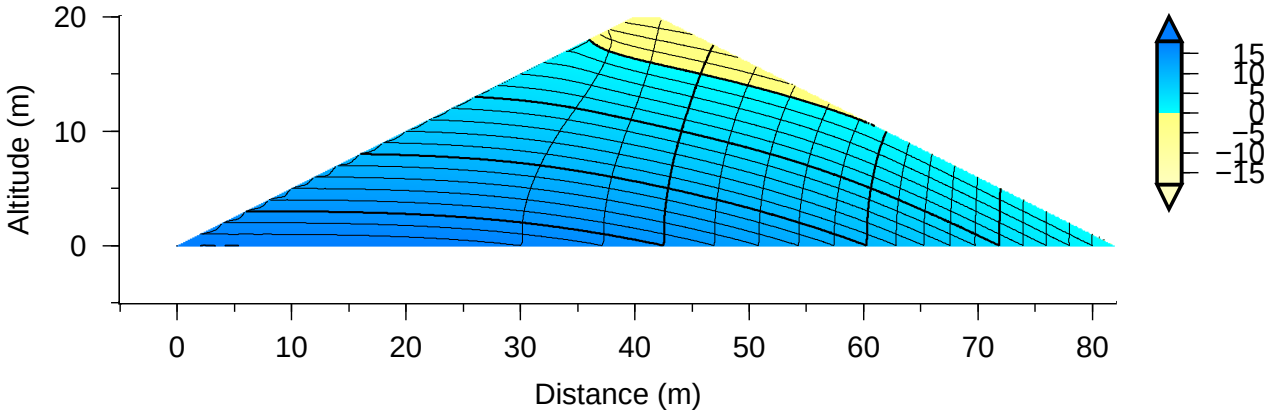


velocity vector map

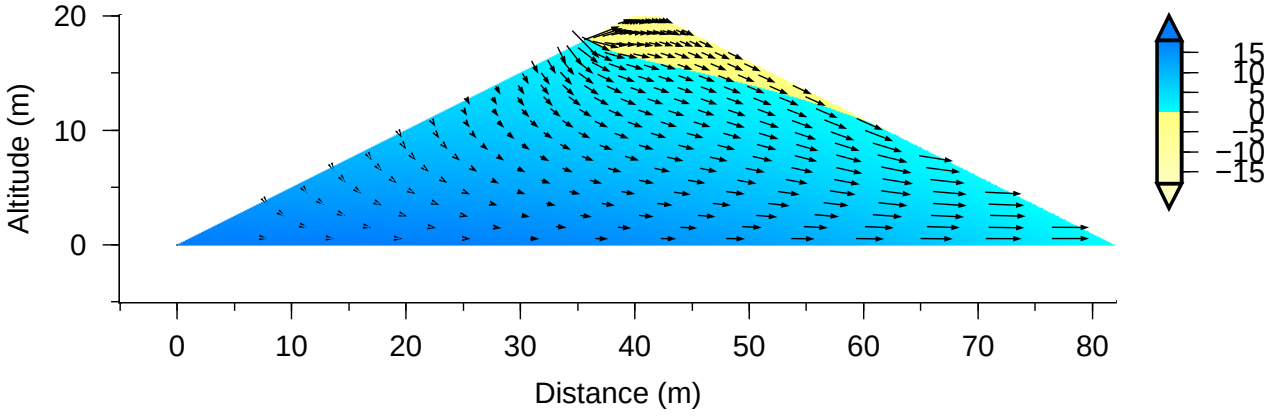
Fig.6 Output sample by 3 node element (slope 1:2.0)



Mesh

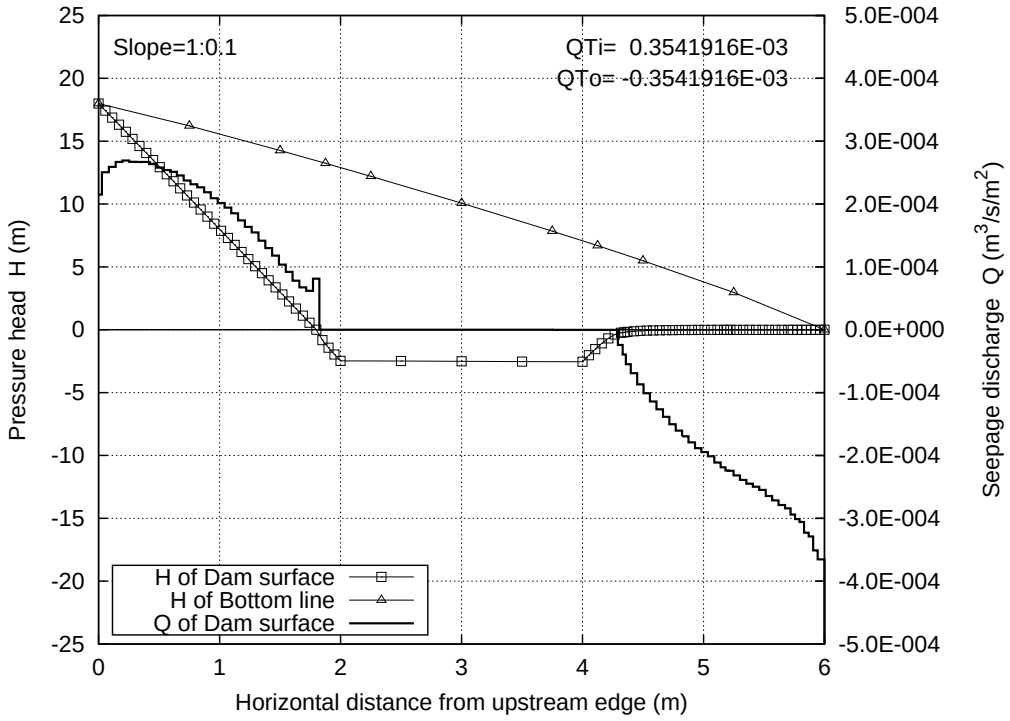


Pressure head contour map

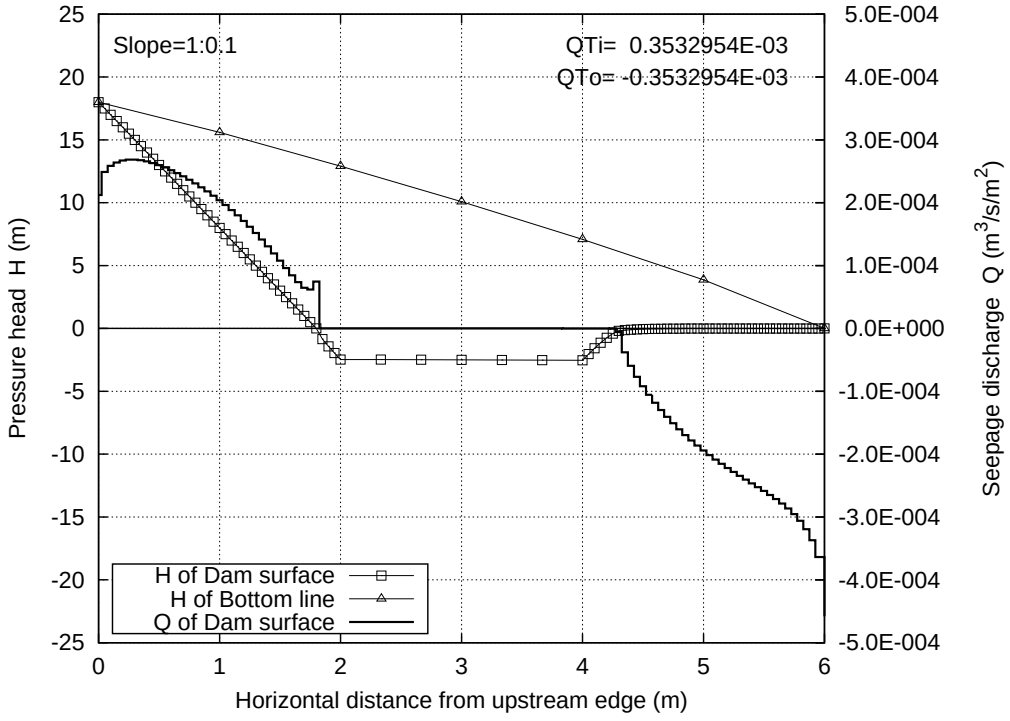


Velocity vector map

Fig.7 Output sample by 4 node element (slope 1:2.0)

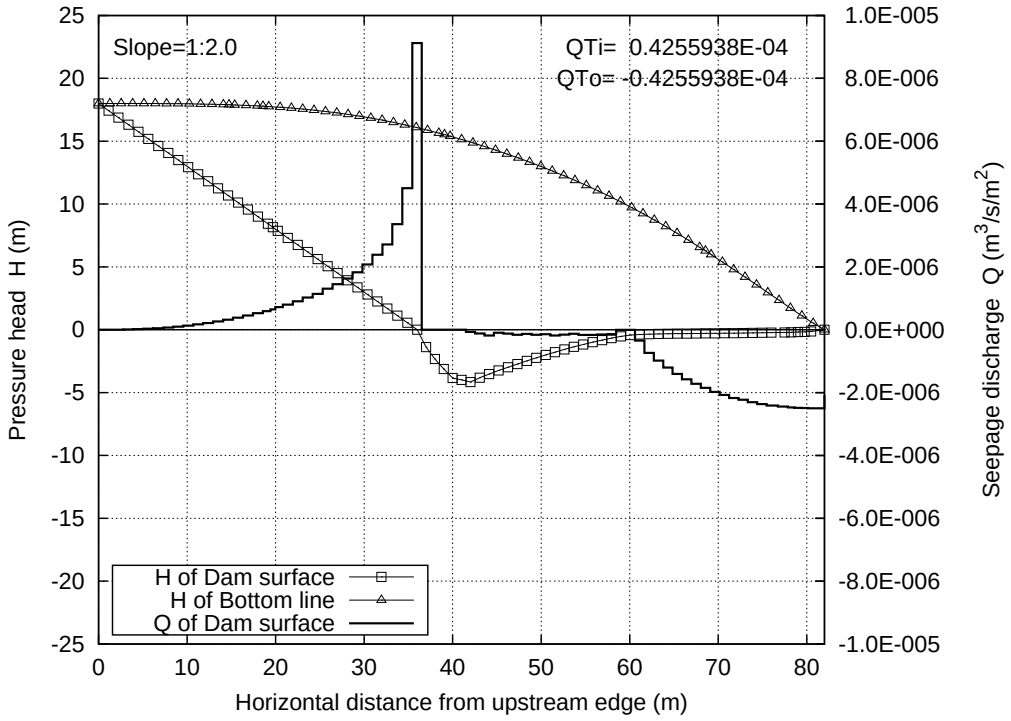


3 node element

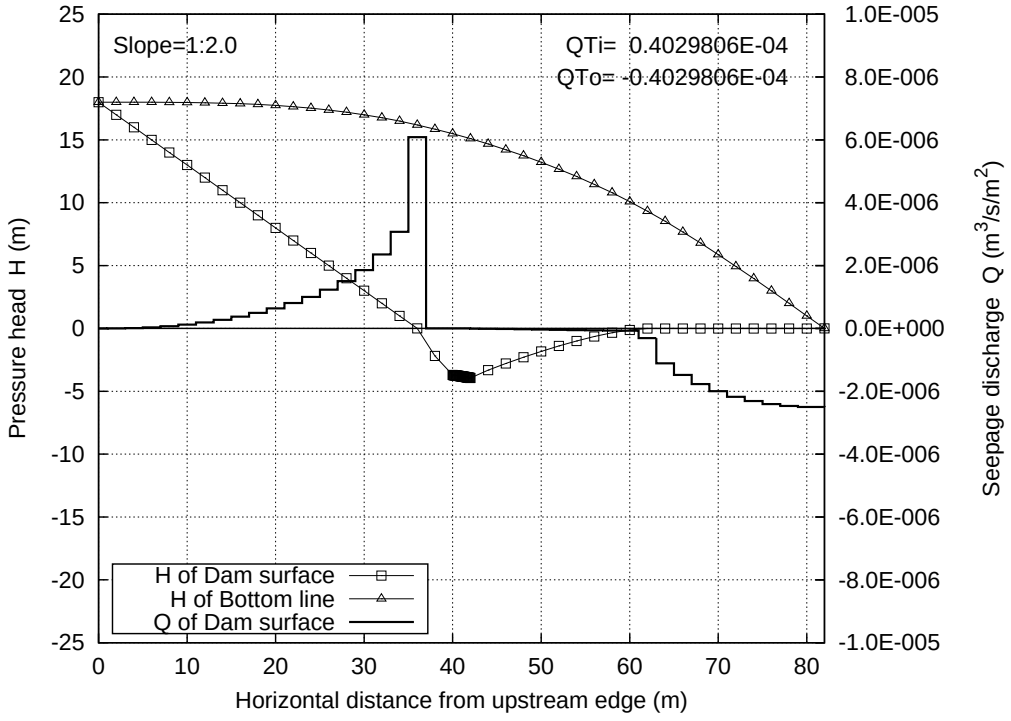


4 node element

Fig.8 Pressure head and discharge distribution (slope 1:0.5)



3 node element



4 node element

Fig.9 Pressure head and discharge distribution (slope 1:2.0)

2. Zoned embankment dam

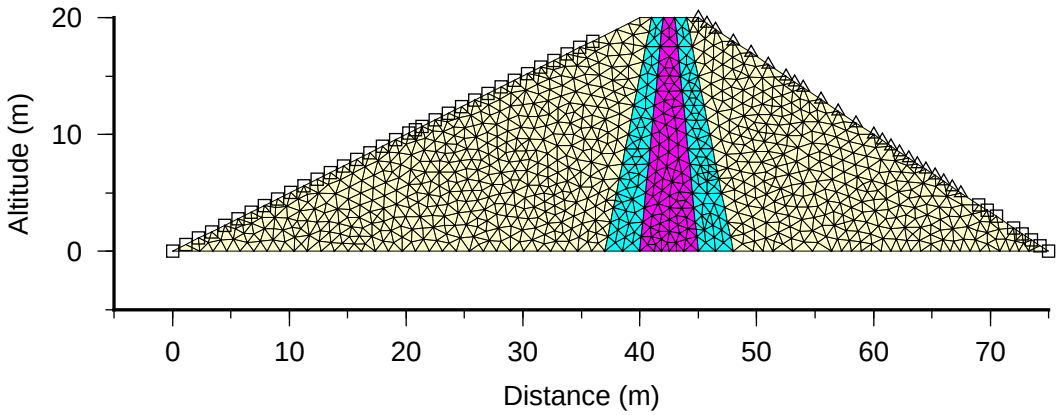
In this section, examples of analysis for zoned embankment dam are shown. In addition, it is a well-known fact for zoned embank dam that the free water surface serves as an action which has a permeating point on the core back and does not become such form shown in this analysis.

Table3 Conditions of analysis

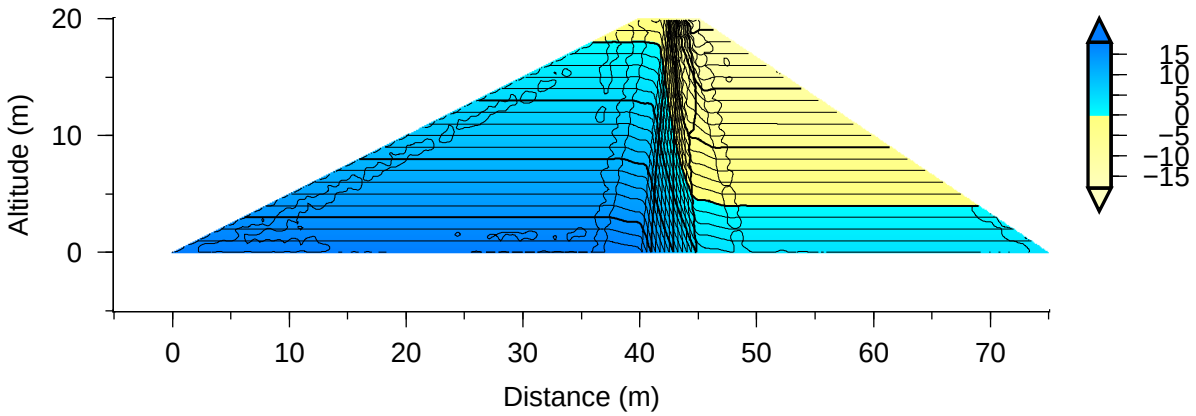
Height (m)	Top width (m)	Upstream slope	Downstream slope	Water depth of upstream (m)	water depth of downstream (m)
20.0	5.0	1:2.0	1:1.5	18.0	4.0

Material properties

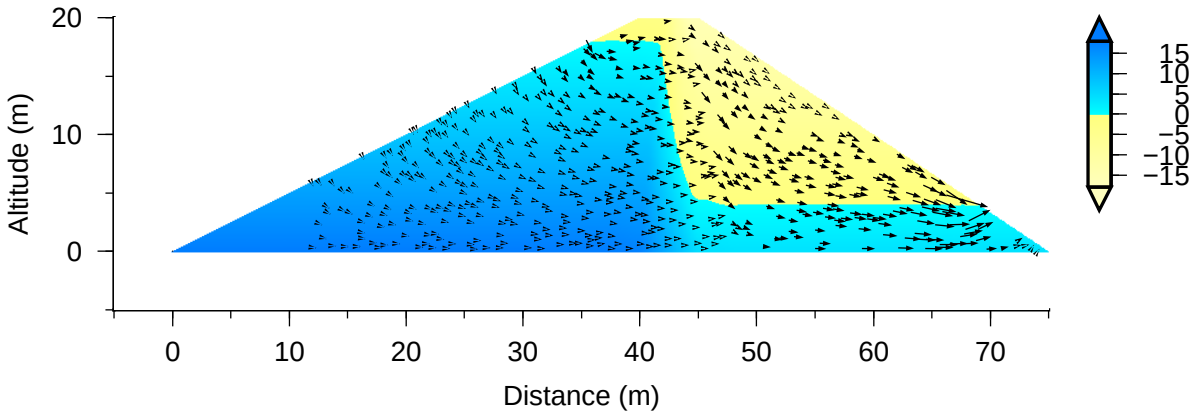
Zone	Permeability k (m/s)	α (m^{-1})	m
Upstream shell	1×10^{0}	0.1	0.7
Upstream filter	1×10^{-4}	0.1	0.7
center core	1×10^{-5}	0.1	0.7
Downstream filter	1×10^{-4}	0.1	0.7
Downstream shell	1×10^{0}	0.1	0.7



Mesh

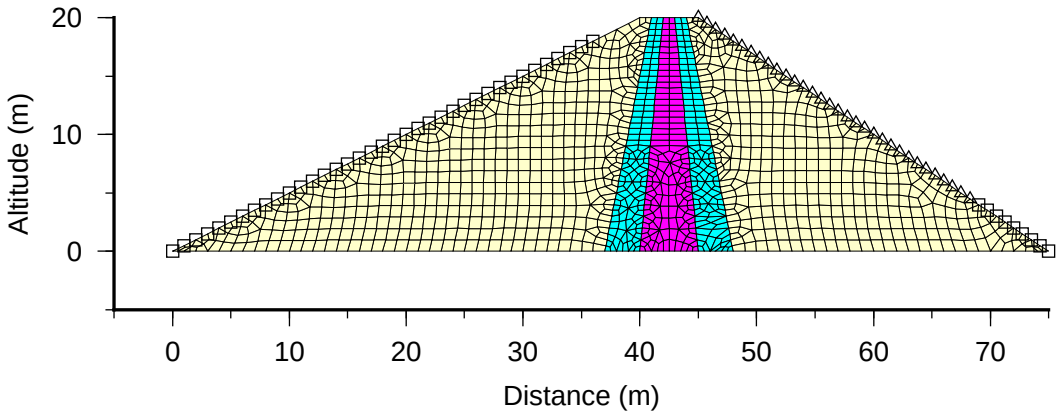


Pressure head contour map

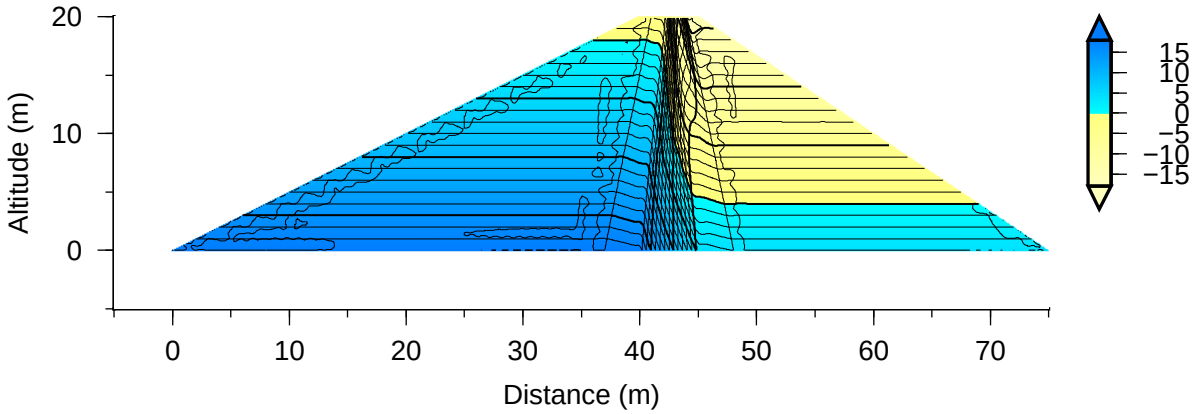


velocity vector map

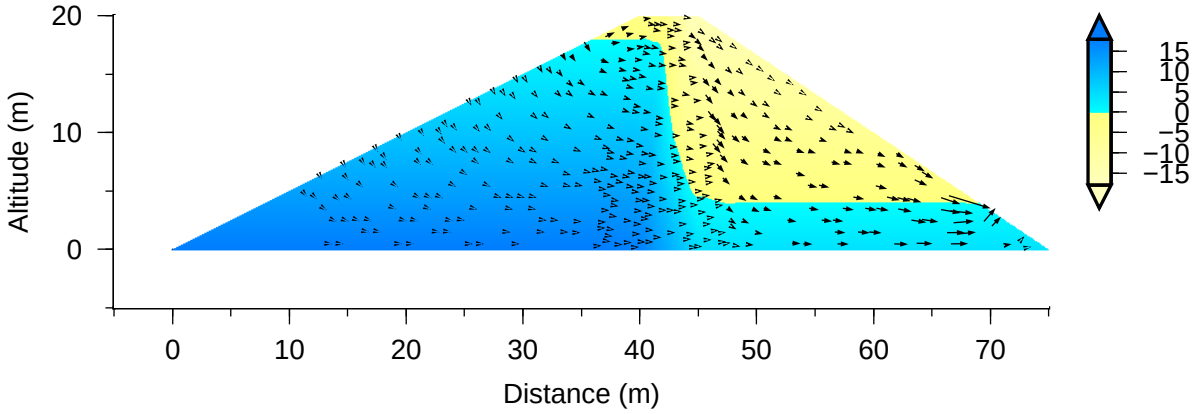
Fig.10 Zoned embankment dam (3 node element)



mesh

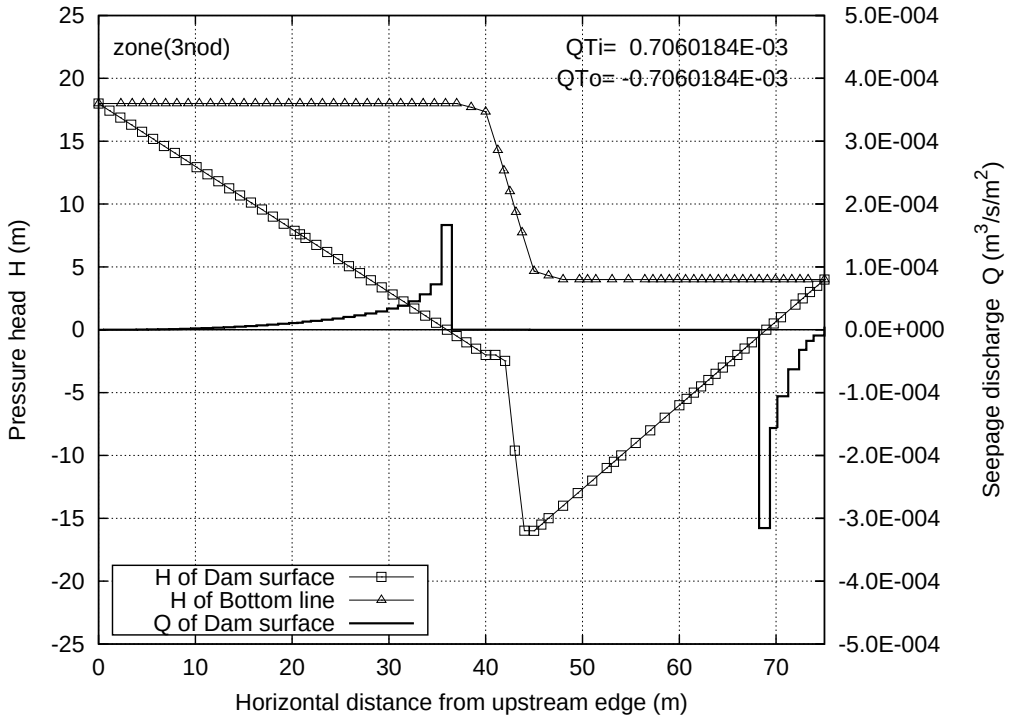


Pressure head contour map

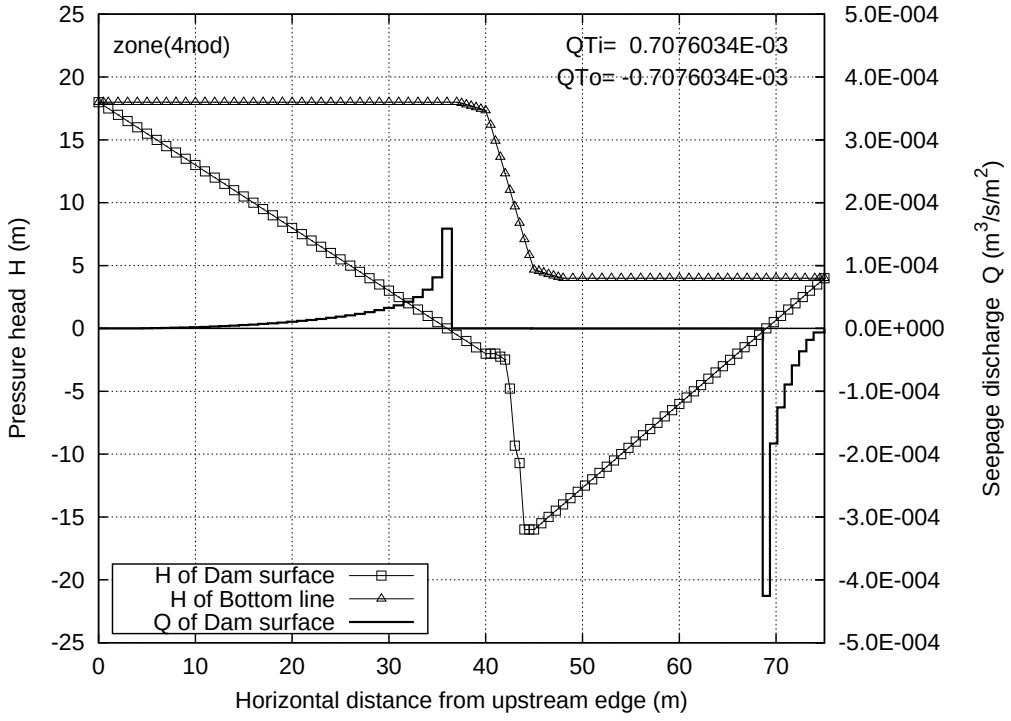


velocity vector map

Fig.11 Zoned embankment dam (4 node element)



3 node element



4 node element

Fig.12 Zoned embankment dam