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Design of Restricted Orifice Surge Tank

Contents

1. Necessity of Surge Tank

In general, a long pressure tunnel, the headrace / tailrace of pumped storage power plant needs a surge tank. The necessity of surge tank is judged whether length of pressure tunnel exceeds 500m or not. A surge tank has following functions.

- ◯ To absorb abnormal pressure increase in the tunnel by water hammering when load rejection or input power rejection.
- ◯ To absorb the abnormal pressure decrease in the tunnel by supplying water when rapid increase of discharge.

Surge tanks are categorized in following 4 types.

- ◯ Simple surge tank
- ◯ Differential surge tank
- ◯ Restricted orifice surge tank (An orifice of a surge tank is called a "port")
- ◯ Surge tank with chamber

These days Restricted Orifice Surge Tanks are usually adopted. This type can effectively attenuate amplitude of the water level in the tank and has comparatively simple design. In addition, in the case that a surge tank is constructed deep under the ground, such as a tailrace surge tank, the diameter of vertical shaft can be reduced by adding upper chamber.

2. Requirements for Hydraulic Design of Restricted Orifice Surge Tank

The following conditions are required for the design of Restricted Orifice Surge Tank.

- ◯ The maximum amplitude of water level by surging is between the elevation of top and bottom of the tank.
- ◯ To meet the required stability conditions (dynamic and static conditions) of water level amplitude.
- ◯ Maximum discharge does not exceed the following critical discharge.

(Critical Discharge)

- ◯ Critical Discharge is defined as discharge when amount of water rise at partial load rejection exceeds that at full load rejection in a restricted orifice surge tank.
- ◯ If the maximum discharge is smaller than the critical discharge, the amount of water rise in the tank at full load rejection is the largest.
- ◯ Although it is quit the thing to confirm the highest level of water rise at the critical discharge in the case that the maximum discharge exceeds the critical discharge, in practical, a surge tank is designed so that maximum discharge does not exceed critical discharge.

3. Fundamental Differential Equations

3.1 Equation of Motion

Euler's equation of fluid motion in one-dimensional problem is as follows.

$$\frac{dv}{dt} = -\frac{1}{\rho}\frac{dp}{dx} = -g\frac{dh}{dx}$$

where	v	: Flow velocity	t	: Time	
	ρ	: Density of the water	p	: Pressure	
	x	: Position	h	: Water head	
	g	: Acceleration of gravity			

When the flow direction from reservoir to surge tank is positive, and the direction of water surface movement from up to dawn on the basis of reservoir water level in a surge tank is positive, positive gradient of water head is upward movement of water level in the surge tank.

$$\frac{dv}{dt} = -g\frac{dh}{dx} = g\frac{z}{L}$$

where z : Variation of the water level in surge tank
(positive is the downward as standard at reservoir water level)
L : Length of the pressure tunnel
v : Flow velocity of the pressure tunnel

If head loss Δh exists, the acceleration of the water in the pressure tunnel is reduced by Δh.

$$\frac{dv}{dt} = g\frac{z}{L} - g\frac{\Delta h}{L} = \frac{z - \Delta h}{L/g}$$

Because friction loss ($h = c{*}v{*}v$) in the pressure tunnel and resistance loss of the port ($k = v_p{*}v_p/2/g$) are caused in the case of Restricted Orifice Surge Tank, the equation of motion is given as follows taking into consideration the flow direction.

$$\frac{dv}{dt} = \frac{z - \Delta h}{L/g} = \frac{z - c \cdot |v| \cdot v - k}{L/g}$$

3.2 Equation of Continuity

When all flow direction of tunnel discharge, discharge in a surge tank and discharge of turbine is assumed positive, equation of continuity is given as follows.

$$Q = f \cdot v + F\frac{dz}{dt} \quad \rightarrow \quad \frac{dz}{dt} = \frac{Q - f \cdot v}{F}$$

where Q : Discharge of the turbine
f : Area of the pressure tunnel
F : Area of the surge tank shaft

3.3 Resistance of Port

Because the port velocity (v_p) is what discharge in a surge tank is divided by the effective area of the port, the resistance of the port (k) can be given as follows, considering the flow direction.

$$v_p = \frac{Q - f \cdot v}{C_d \cdot F_p} \quad \rightarrow \quad k = \frac{|v_p| \cdot v_p}{2g} = \frac{1}{2g}\left|\frac{f \cdot v - Q}{C_d \cdot F_p}\right| \cdot \frac{f \cdot v - Q}{C_d \cdot F_p}$$

3.4 Fundamental differential equations

Equation of motion : $$\frac{dv}{dt} = \frac{z - c \cdot |v| \cdot v - k}{L/g} \tag{1}$$

Continuous equation : $$\frac{dz}{dt} = \frac{Q - f \cdot v}{F} \tag{2}$$

Resistance of the port : $$k = \frac{|v_p| \cdot v_p}{2g} = \frac{1}{2g} \cdot \left|\frac{f \cdot v - Q}{C_d \cdot F_p}\right| \cdot \frac{f \cdot v - Q}{C_d \cdot F_p} \tag{3}$$

v_p : Flow velocity of the port
F_p : Area of the port
C_d : Discharge coefficient of the port
v : Flow velocity of the pressure tunnel (positive is from reservoir to surge tank)
z : Variation of the water level in surge tank
g : Acceleration of gravity
c : Head loss coefficient
L : Length of the pressure tunnel (from reservoir to the port)
f : Area of the pressure tunnel
F : Area of the shaft
Q : Discharge (in the interception current of the time)

4. Equations for the basic design of Restrict Orifice Surge Tank

4.1 Formula to calculate the maximum water level in the tank

(1) Vogt-Forchheimer's Formulas

Vogt-Forchheimer's Formulas are used to calculate the highest rising water level in the basic design stage. This is the one rewritten to be convenient equation after solving the fundamental differential equation on the condition of instant intercept of initial discharge.

Vogt-Forchheimer's formulas

$$\begin{cases} m' \cdot k_0 < 1 & (1 + m' \cdot z_m) - \ln(1 + m' \cdot z_m) = (1 + m' \cdot h_0) - \ln(1 - m' \cdot k_0) \\ m' \cdot k_0 > 1 & (m' \cdot |z_m| - 1) + \ln(m' \cdot |z_m| - 1) = \ln(m' \cdot k_0 - 1) - (m' \cdot h_0 + 1) \end{cases} \tag{4}$$

$$h_0 = c \cdot {v_0}^2$$
$$k_0 = \frac{1}{2g}\left(\frac{Q_0}{C_d F_p}\right)^2$$
$$m' = \frac{2gF(h_0 + k_0)}{Lf{v_0}^2}$$

z_m	: Maximum water level in the tank (Positive is the downward as astandard at reservoir level)
h_0	: Total head loss of the pressure tunnel
k_0	: Resistance of the port
v_0	: Flow velocity of the pressure tunnel
c	: Head loss coefficient ($h_0 = c \cdot {v_0}^2$)
Q_0	: Maximum discharge
F_p	: Area of the port
C_d	: Discharge coefficient of the port
L	: Length of the pressure tunnel (from reservoir to the port)
f	: Area of the pressure tunnel
F	: Area of the shaft
g	: Acceleration of gravity

To calculate (z_m) in the formula above, Newton-Raphson Method is leveraged. z which makes $f(z) = 0$ is the maximum water level (z_m) in the function of $f(z)$ and $f'(z)$. Here, $f'(z)$ is a derived function of $f(z)$.

$$f(z) = \begin{cases} \{(1 + m' \cdot z) - \ln(1 + m' \cdot z)\} - \{(1 + m' \cdot h_0) - \ln(1 - m' \cdot k_0)\} & (m' \cdot k_0 < 1) \\ \{m' \cdot |z| - 1) + \ln(m' \cdot |z| - 1)\} - \{\ln(m' \cdot k_0 - 1) - (m' \cdot h_0 + 1)\} & (m' \cdot k_0 > 1) \end{cases}$$

$$f'(z) = \begin{cases} m'\left(1 - \dfrac{1}{1 + m'z}\right) & (m' \cdot k_0 < 1) \\ m'\left(1 + \dfrac{1}{m'|z| - 1}\right) & (m' \cdot k_0 > 1) \end{cases}$$

Calculation of the below equation is iterated until $f(z_i + 1)$ becomes nearly equal 0.

$$z_{i+1} = z_i - \frac{f(z_i)}{f'(z_i)}$$

Initial value z_0 is defined as below so that the value in the logarithm paragraph of $f(z)$ can be positive.

$$\begin{cases} z_0 = -\dfrac{1}{m'} + 0.0001 & (m' \cdot k_0 < 1) \\ |z_0| = \dfrac{1}{m'} + 0.0001 & (m' \cdot k_0 > 1) \end{cases}$$

(2) Required conditions to calculate water level of reservoir and head loss

To calculate water level, following issues are considered.

- ○ To adopt the most rigid condition of water level of the reservoir corresponding to the discharge conditions.
- ○ To set the roughness small in the case of load rejection or input power rejection, and to set the roughness large in the case of rapid increase of discharge on the safe side.

Table1 Conditions of the water level and roughness

Discharge	Water level and roughness	Headrace surge tank	Tailrace surge tank
Total Load interception	Reservoir water level	HWL of Upper Res.	LWL of Lower Res.
	Checked water level	Upper surge W.L.	Down surge W.L.
	Variation of roughness	-0.0015	-0.0015
Load Rapidly increase	Reservoir water level	LWL of Upper Res.	HWL of Lower Res.
	Checked water level	Down surge W.L.	Upper surge W.L.
	Variation of roughness	$+0.0015$	$+0.0015$
Total Input interception	Reservoir water level	LWL of Upper Res.	HWL of Lower Res.
	Checked water level	Down surge W.L.	Upper surge W.L.
	Variation of roughness	-0.0015	-0.0015

Notes) Variation of roughness is for the concrete lining.

(Supplemental Explanation)

○ The roughness of concrete to calculate surging water level are set by adding the value above or subtracting it from the normal value of 0.013∼0.0125.

○ In the case of steel lining, 0.001, and in the case of no lining, 0.003 is added or subtracted from the normal value to set the roughness respectively.

4.2 Requirement for Stability of Water Level Vibration

Thoma-Schüller's formulas

Static stability conditions : $$h_0 < \frac{H_g}{3} \sim \frac{H_g}{6} \tag{5}$$

Dynamic stability conditions : $$F > \frac{Lf}{c(1+\eta)gH_g} \sim \frac{Lf}{2cg(H_g - z_m)} \tag{6}$$

$$\eta = \frac{k_0}{h_0} \qquad h_0 = c \cdot {v_0}^2 \qquad k_0 = \frac{1}{2g}\left(\frac{Q_0}{C_d F_p}\right)^2$$

H_g : Gross head

z_m : Maximum water level as a standard at reservoir water level

4.3 Equation of Critical Discharge

To calculate the critical discharge Q_c , Calame-Garden equation is leveraged.

$$Q_c = \frac{1}{c}\left(\frac{1}{2g} \cdot \frac{Lf^3}{F\eta}\right)^{1/2} \tag{7}$$

5. Procedure of Basic Design of Surge Tank

5.1 Correction of Head Loss

Head loss coefficient (c) is calculated by using the head loss of intake and headrace for a headrace surge tank, the head loss of tailrace and outlet for a tailrace surge tank, taking into consideration correction of head loss according to **Table 1**. The representative velocity of headrace / tailrace can be used as v_0 in the calculation.

5.2 Check of Static Stability

Requirement of static stability is determined by only total water head H_g as shown in the equation (5). Therefore, if this requirement is not satisfied, head loss should be reduced by increasing of tunnel diameter and so on.

5.3 Set Targeted Movement Range of Water Level

The targeted movement range of water level is set corresponding to the low water level of the reservoir and bottom elevation of the tank. Since surging is attenuating vibration, even though full load rejection while reservoir water level is low water level is taken place, the drawdown depth does not exceed the rise up depth. However, in practical, the targeted movement range of water level is set as both depths are the almost same.

5.4 Relationship between Port Diameter and Shaft Diameter

The available ranges of the port diameter and the shaft diameter, which are calculated from the requirement of dynamic stability and critical discharge, are to be set. Relationship between the port diameter and shaft diameter is given by following expressions.

$$F > F_1 = \frac{h_0 L f}{c(h_0 + k_0) g H_g} \qquad \text{from equation (6), 1st term of right side} \tag{8}$$

$$F > F_2 = \frac{L f}{2cg(H_g - z_m)} \qquad \text{from equation (6), 2nd term of right side} \tag{9}$$

$$F < F_3 = \frac{h_0 L f^3}{2gc^2 k_0 {Q_0}^2} \qquad \text{from the condition } Q_0 < Q_c \text{ in equation (7)} \tag{10}$$

(Supplementary Explanation)

○ Necessary ranges of the port diameter and shaft diameter are found by equation (8) and (10).

○ Minimum value of the shaft diameter is found by substituting the targeted movement range of water level to zm in the equation (9).

○ Since requirement of equation (9) is usually more rigid than that of equation (8), the shaft diameter needs to be larger than the value found by equation (9).

5.5 Relationship between Port Diameter, Shaft Diameter and Maximum Water Level

The correlation between the port diameter and maximum upper water level by the equation (4) is calculated by making shaft diameter a parameter within the range which meets the requirement of dynamic stability. The correlation between the port diameter and resistance loss of port at the load rejection and input power rejection is calculated.

$$|z'_{rm}| = k_0 - h_0 \qquad (|z'_{rm}| : \text{Resistance loss of the port at the cut-off})$$

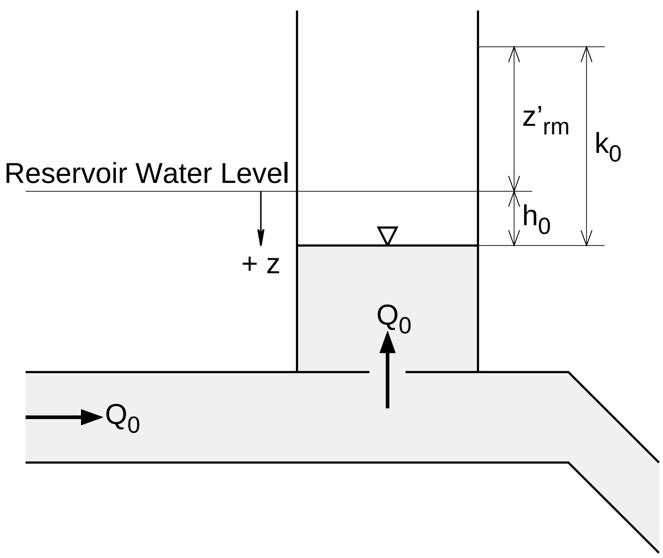
5.6 Selection of the shaft and port diameter

The optimal port diameter is found so that the resistance loss of the port —z'rm— at the load rejection is equal to the maximum upper water level $|z_m|$.

$$|z'_{rm}| = |z_m| \qquad \text{(Condition of optimal port diameter)}$$

Therefore, the shaft diameter and the port diameter are determined to meet the above condition and requirement of within the targeted fluctuation range.

(Supplementary Explanation)



The tank water level before interception is lowered only head loss h_0 of the pressure tunnel from the reservoir water level. If discharge Q_0 before interception tends to flow in in a tank from the port at the moment of interception, the velosity head at the port tends to become $k_0 = \frac{1}{2g}(Q_0/C_d/F_p)^2$, and tends to push up the tank water level with this energy (where C_d is the discharge coefficient of the port, and F_p is the area of the port). Therefore, the amount of the water level rise from the reservoir water level becomes $|z'_{rm}| = k_0 - h_0$. The absolute value is taken because "positive is the downward" on the basis of the reservoir water level.
Here, evaluation of the highest rise water level is performed as follows.

$|z_m| < |z'_{rm}|$: diameter of the port is estimated as a small state because of large resistance of the port.
$|z_m| = |z'_{rm}|$: diameter of the port is the optimal.
$|z_m| > |z'_{rm}|$: diameter of the port is estimated as a large state because of small resistance of the port.
where $|z_m|$ is calculated value from the Vogt-Forchheimer formula.

In addition, the pressure head of the bottom of surge tank is denoted by the sum of the pressure head of water in the tank, and resistance of the port k. For this reason, in the structural design of the bottom of surge tank, it is necessary to carry out in consideration of the additional pressure head k.

5.7 Check of Dynamic Stability

It should be checked that the selected shaft diameter and port diameter meet the requirement of dynamic stability and critical discharge, by plotting selected value on the figure of correlation between dynamic stability and critical discharge.

5.8 Arrangement of Chamber

Excavation volume of the shaft can be reduced by adding a chamber for a surge tank which is constructed in deep underground, such as a tailrace surge tank. The figure of the chamber is designed so that the capacity of chamber can absorb the water volume of upper surge over the level of chamber bottom which is calculated by equation (4) in the case of shaft type surge tank.

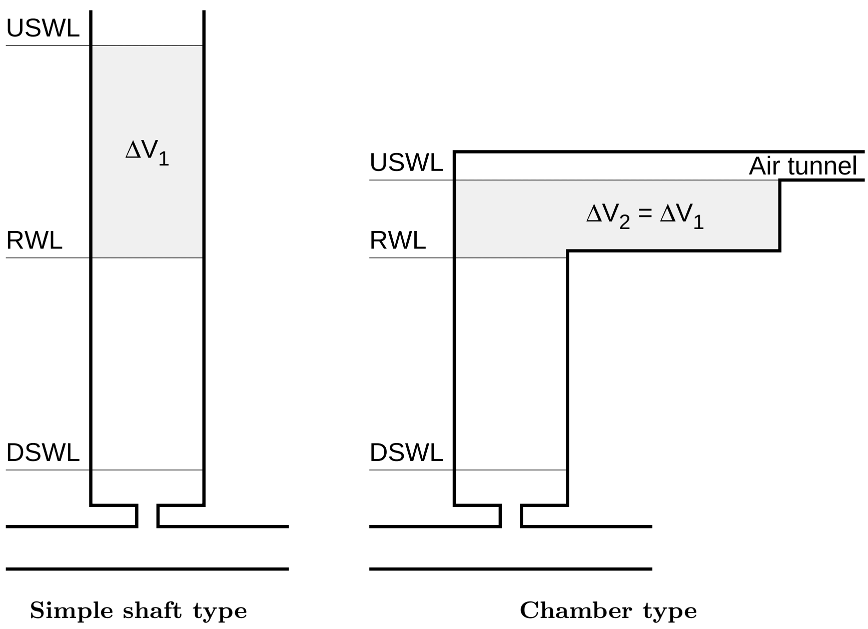


Fig.1 Design concept of Chamber type

6. Surging analysis

The design of the surge tank is checked by surging analysis. Chronological movement of water level in the tank is found by solving differential equations of (1), (2), (3).

7. Examples of Surging Analysis

Analysis of Headrace surge tank and Tailrace surge tank

Table2 Conditions of the calculation for the stability and maximum water lebel

Items		Headrace surge tank		Tailrace surge tank	
		Load interception	Input interception	Load interception	Input interception
H_g	(m)	713		713	
L	(m)	2553.370		2167.752	
d_0	(m)	8.2		8.2	
C_d	–	0.9		0.9	
z_m (target value)	(m)	35		65	
Q_0	(m^3/s)	340	240	340	240
c	–	0.179	0.185	0.166	0.160
Decided Shaft dia.	(m)	17.0		10.0	
Decided Port dia.	(m)	4.6		4.9	

d_0 is the diameter of the pressure tunnel.

$$c = \frac{h_0}{v_0^2}$$

c : Head loss coefficient

h_0 : Total head loss of the pressure tunnel

v_0 : Flow velocity of the pressure tunnel

Table3 Conditions for the surging analysis

Items	Headrace surge tank			Tailrace surge tank		
	EL. (m)	Area (m^2)	Shape (m)	EL. (m)	Area (m^2)	Shape (m)
Top of Surge tank	1566.000	226.980	ϕ17.0	865.000	520.000	□13x40
Bottom of Chamber	–	–	–	854.050	520.000	□13x40
Bottom of Surge tank	1461.600	226.980	ϕ17.0	727.600	778.540	ϕ10.0
Port	–	16.619	ϕ4.6	–	18.857	ϕ4.9
Case	①	②	③	①	②	③
R.W.L (m)	1527	1500	1500	814	844	844
c (loss coefficient)	0.179	0.253	0.185	0.166	0.277	0.160
Discharge (m^3/s)	340 to 0	170 to 340	−240 to 0	−340 to 0	−170 to −340	240 to 0
Interception time (sec)	8.0	40.0	5.6	8.0	40.0	5.6

Case ① : Load interception (4 units)
Case ② : Load rapidly increase (4 units)
Case ③ : Input interception (4 units)
"R.W.L" is the reservoir water level.
Discharge "+" is the direction from Reservoir to Surge tank.

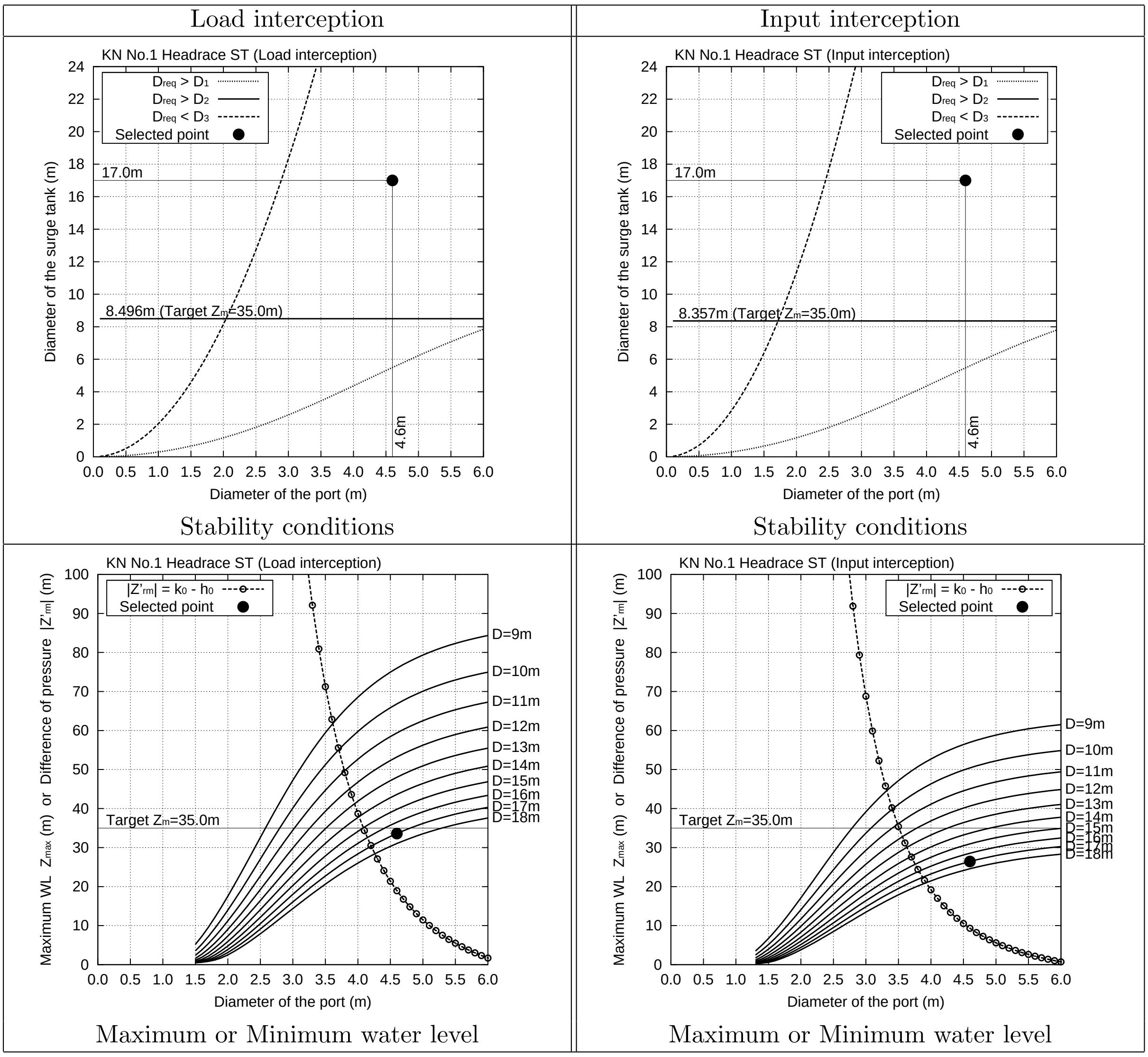


Fig.2 Headrace Surge tank

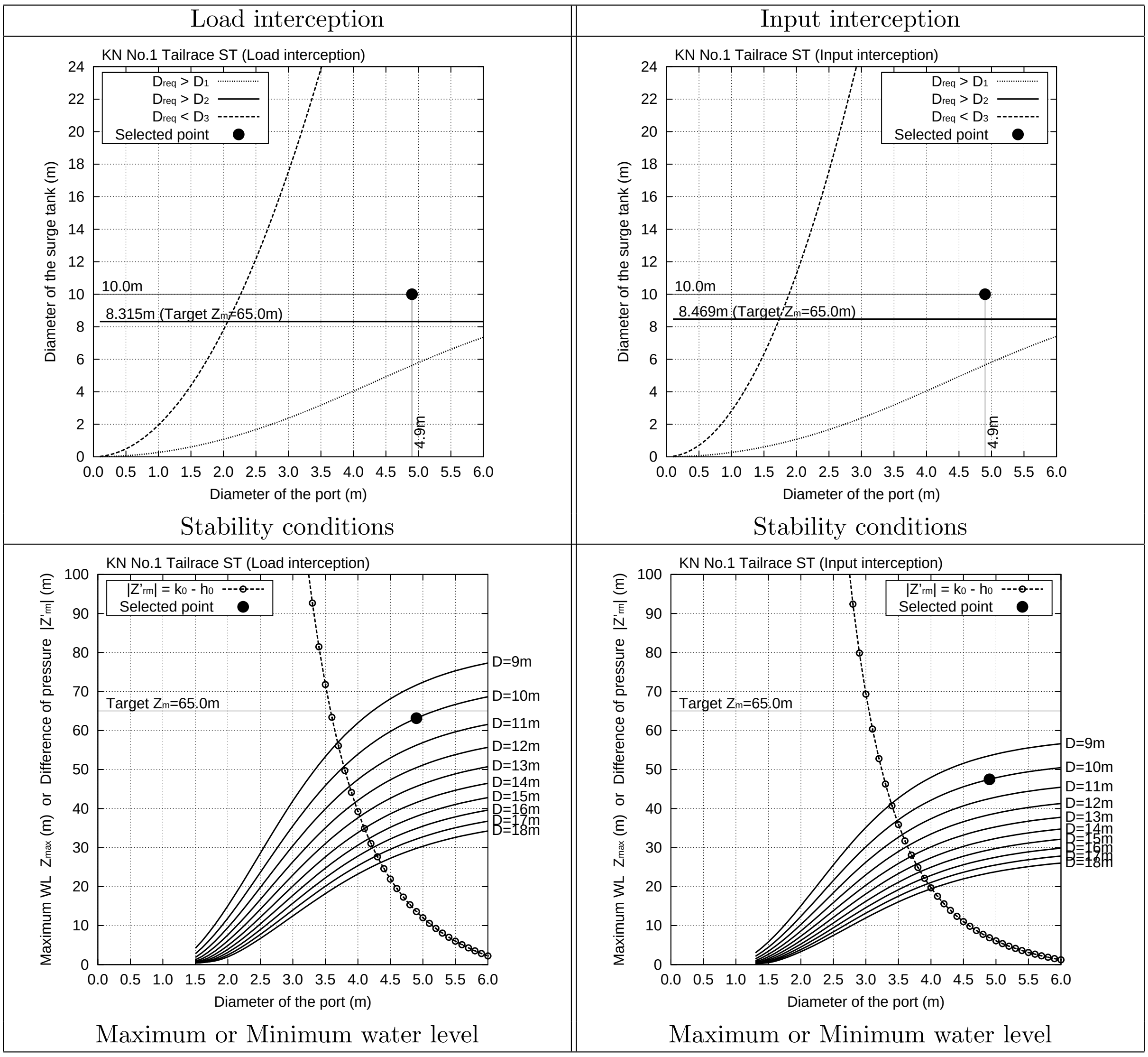


Fig.3 Tailrace Surge tank

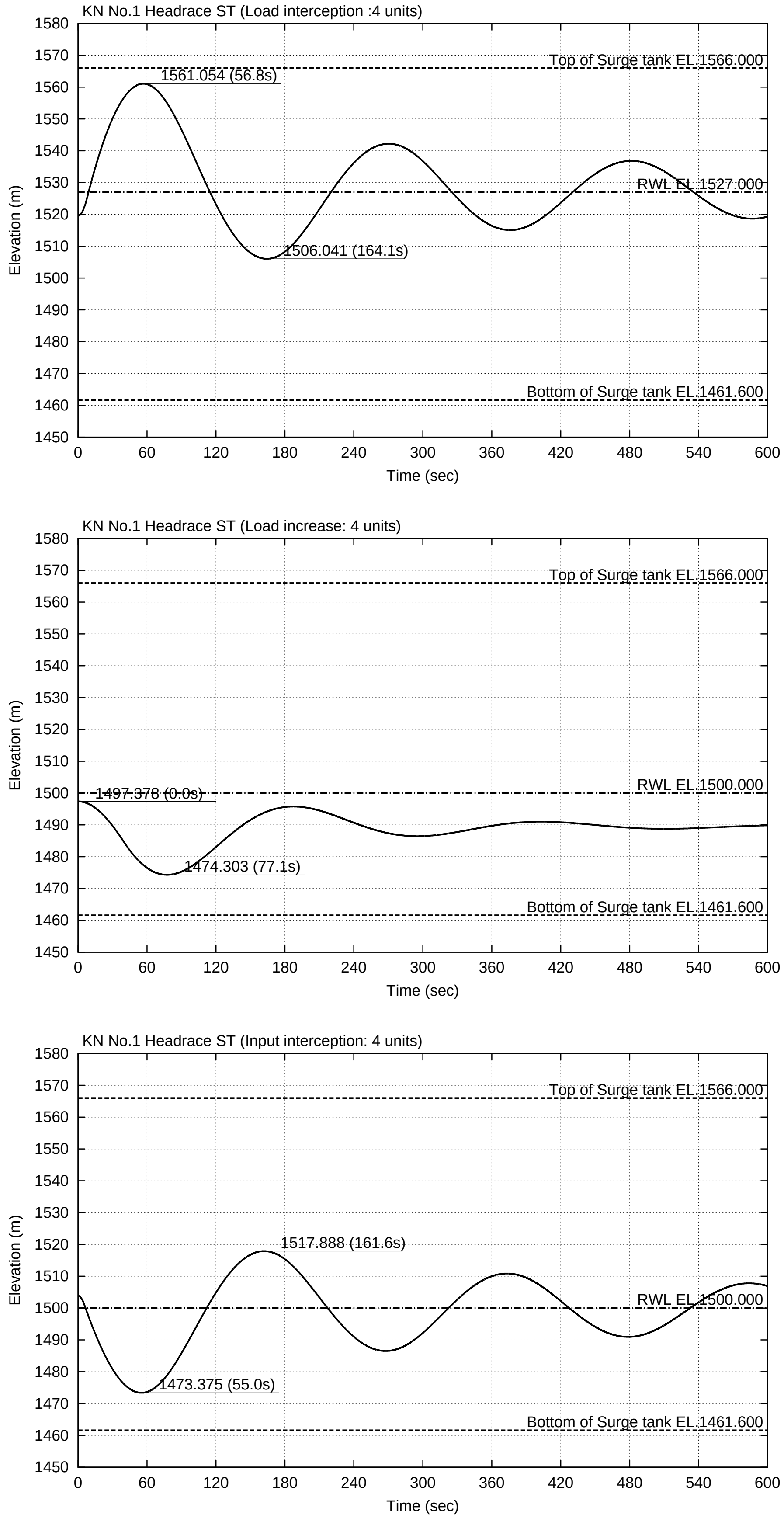


Fig.4 Results of the surging analysis for Headrace surge tank

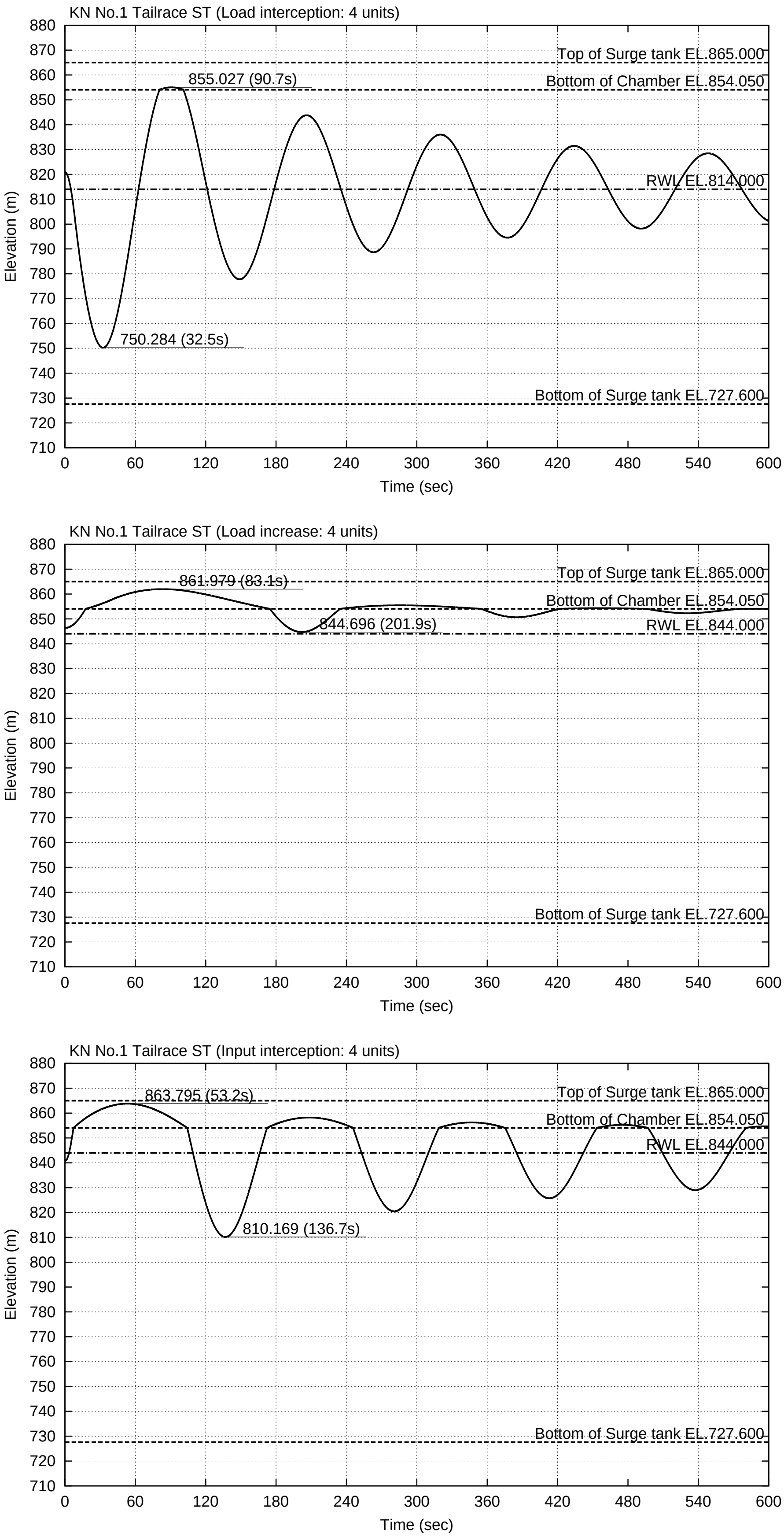


Fig.5 Results of the surging analysis for Tailrace surge tank