

Basics of Vibration

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1. Numerical Integration of Single Degree of Freedom (SDOF) system

Numerical integration methods of the equation of motion of SDOF system to get the acceleration, velocity and displacement are introduced.

1.1 Equation of Motion

The equation of motion of SDOF system at instance $t + \Delta t$ is shown below, where dot means differentiation by time t .

$$m \cdot \ddot{u}(t + \Delta t) + c \cdot \dot{u}(t + \Delta t) + k \cdot u(t + \Delta t) = f(t + \Delta t) \quad (1)$$

where, m : mass \ddot{u} : acceleration f : external force
 c : damping \dot{u} : velocity
 k : stiffness u : displacement

1.2 Linear acceleration method

(1) Displacement

Use Taylor series for the displacement u at $t + \Delta t$ and consider the terms from 1st to 4th of the right side.

$$u(t + \Delta t) = u(t) + \frac{\Delta t}{1!} \cdot \dot{u}(t) + \frac{(\Delta t)^2}{2!} \cdot \ddot{u}(t) + \frac{(\Delta t)^3}{3!} \cdot \dddot{u}(t) + \dots \quad (2)$$

$$= u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{3} \cdot \ddot{u}(t) + \frac{(\Delta t)^2}{6} \cdot \ddot{u}(t + \Delta t) \quad (3)$$

In the linear acceleration method, the differential coefficient of 3rd order of the displacement u is linearized using following equation:

$$\dddot{u}(t) = \frac{\ddot{u}(t + \Delta t) - \ddot{u}(t)}{\Delta t} \quad (4)$$

(2) Velocity

Using a trapezoidal formula, following form can be obtained.

$$\dot{u}(t + \Delta t) = \dot{u}(t) + \int_t^{t+\Delta t} \ddot{u}(t) dt = \dot{u}(t) + \frac{\Delta t}{2} \cdot (\ddot{u}(t + \Delta t) + \ddot{u}(t)) \quad (5)$$

(3) Acceleration

Substituting the displacement and velocity which are derived from above to the equation of motion, following form to calculate the acceleration can be obtained.

$$\ddot{u}(t + \Delta t) = \frac{f(t + \Delta t) - c \cdot \left(\dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t) \right) - k \cdot \left(u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{3} \cdot \ddot{u}(t) \right)}{m + c \cdot \frac{\Delta t}{2} + k \cdot \frac{(\Delta t)^2}{6}} \quad (6)$$

(4) Summary

Linear acceleration method

$$\ddot{u}(t + \Delta t) = \frac{f(t + \Delta t) - c \cdot \left(\dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t) \right) - k \cdot \left(u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{3} \cdot \ddot{u}(t) \right)}{m + c \cdot \frac{\Delta t}{2} + k \cdot \frac{(\Delta t)^2}{6}} \quad (7)$$

$$\dot{u}(t + \Delta t) = \dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t + \Delta t) \quad (8)$$

$$u(t + \Delta t) = u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{3} \cdot \ddot{u}(t) + \frac{(\Delta t)^2}{6} \cdot \ddot{u}(t + \Delta t) \quad (9)$$

1.3 Average acceleration method

(1) Displacement

Use Taylor series for the displacement u at $t + \Delta t$ and consider the terms from 1st to 3rd of the right side.

$$u(t + \Delta t) = u(t) + \frac{\Delta t}{1!} \cdot \dot{u}(t) + \frac{(\Delta t)^2}{2!} \cdot \ddot{u}(t) + \dots \quad (10)$$

$$= u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{4} \cdot \ddot{u}(t) + \frac{(\Delta t)^2}{4} \cdot \ddot{u}(t + \Delta t) \quad (11)$$

In the average acceleration method, the differential coefficient of 2nd order of the displacement u is linearized using following equation:

$$\ddot{u}(t) = \frac{\ddot{u}(t + \Delta t) + \ddot{u}(t)}{2} \quad (12)$$

(2) Velocity

Similar to the case of the linear acceleration method, following equation can be obtained:

$$\dot{u}(t + \Delta t) = \dot{u}(t) + \int_t^{t+\Delta t} \ddot{u}(t) dt = \dot{u}(t) + \frac{\Delta t}{2} \cdot (\ddot{u}(t + \Delta t) + \ddot{u}(t)) \quad (13)$$

(3) Acceleration

Substituting the displacement and velocity which are derived from above to the equation of motion, following form to calculate the acceleration can be obtained.

$$\ddot{u}(t + \Delta t) = \frac{f(t + \Delta t) - c \cdot \left(\dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t) \right) - k \cdot \left(u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{4} \cdot \ddot{u}(t) \right)}{m + c \cdot \frac{\Delta t}{2} + k \cdot \frac{(\Delta t)^2}{4}} \quad (14)$$

(4) Summary

Average acceleration method

$$\ddot{u}(t + \Delta t) = \frac{f(t + \Delta t) - c \cdot \left(\dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t) \right) - k \cdot \left(u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{4} \cdot \ddot{u}(t) \right)}{m + c \cdot \frac{\Delta t}{2} + k \cdot \frac{(\Delta t)^2}{4}} \quad (15)$$

$$\dot{u}(t + \Delta t) = \dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t + \Delta t) \quad (16)$$

$$u(t + \Delta t) = u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{4} \cdot \ddot{u}(t) + \frac{(\Delta t)^2}{4} \cdot \ddot{u}(t + \Delta t) \quad (17)$$

1.4 Nermark's β method

(1) Formula for calculation

This method is generalized method of the linear acceleration method. The use of taylor series for the displacement u and velocity \dot{u} at $t + \Delta t$ provides following forms:

$$u(t + \Delta t) = u(t) + \frac{\Delta t}{1!} \cdot \dot{u}(t) + \frac{(\Delta t)^2}{2!} \cdot \ddot{u}(t) + \frac{(\Delta t)^3}{3!} \cdot \dddot{u}(t) + \dots \quad (18)$$

$$\dot{u}(t + \Delta t) = \dot{u}(t) + \frac{\Delta t}{1!} \cdot \ddot{u}(t) + \frac{(\Delta t)^2}{2!} \cdot \dddot{u}(t) + \dots \quad (19)$$

If we deem the 4th term for displacement as $1/3! = \beta$, the 3rd term for velocity as $1/2! = \gamma$, and we truncate higher order terms, above equations are expressed as follows:

$$u(t + \Delta t) = u(t) + \Delta t \cdot \dot{u}(t) + \frac{(\Delta t)^2}{2} \cdot \ddot{u}(t) + \beta \cdot (\Delta t)^3 \cdot \ddot{u}(t) \quad (20)$$

$$\dot{u}(t + \Delta t) = \dot{u}(t) + \Delta t \cdot \ddot{u}(t) + \gamma \cdot (\Delta t)^2 \cdot \ddot{u}(t) \quad (21)$$

Next, similar to the linear acceleration method, we linearize the term of \ddot{u} as follow:

$$\ddot{u}(t) = \frac{\ddot{u}(t + \Delta t) - \ddot{u}(t)}{\Delta t} \quad (22)$$

To arrange the equation of motion using above relationship provides following equation:

$$u(t + \Delta t) = u(t) + \Delta t \cdot \dot{u}(t) + \left(\frac{1}{2} - \beta\right) (\Delta t)^2 \cdot \ddot{u}(t) + \beta (\Delta t)^2 \cdot \ddot{u}(t + \Delta t) \quad (23)$$

$$\dot{u}(t + \Delta t) = \dot{u}(t) + (1 - \gamma) \cdot \Delta t \cdot \ddot{u}(t) + \gamma \cdot \Delta t \cdot \ddot{u}(t + \Delta t) \quad (24)$$

The formulas for Newmark's β method can be expressed as follow by taking into consideration of $\gamma = 1/2$. (Regarding the value of γ , $\gamma = 1/2$ is normally used because of stability of solution.)

Newmark's β method

$$\ddot{u}(t + \Delta t) = \frac{f(t + \Delta t) - c \cdot \left(\dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t)\right) - k \cdot \left(u(t) + \Delta t \cdot \dot{u}(t) + \left(\frac{1}{2} - \beta\right) \cdot (\Delta t)^2 \cdot \ddot{u}(t)\right)}{m + c \cdot \frac{\Delta t}{2} + k \cdot \beta \cdot (\Delta t)^2} \quad (25)$$

$$\dot{u}(t + \Delta t) = \dot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t) + \frac{\Delta t}{2} \cdot \ddot{u}(t + \Delta t) \quad (26)$$

$$u(t + \Delta t) = u(t) + \Delta t \cdot \dot{u}(t) + \left(\frac{1}{2} - \beta\right) \cdot (\Delta t)^2 \cdot \ddot{u}(t) + \beta \cdot (\Delta t)^2 \cdot \ddot{u}(t + \Delta t) \quad (27)$$

where, $\beta = \frac{1}{4}$ in the average acceleration method and $\beta = \frac{1}{6}$ in the linear acceleration method.

(2) Stability of Newmark's β method

For zero damping, Newmark's method is conditionally stable if satisfying follows:

$$\gamma \geq \frac{1}{2} \quad \beta \leq \frac{1}{2} \quad \frac{\Delta t}{T_{min}} \leq \frac{1}{2\pi\sqrt{\gamma/2 - \beta}} \quad (28)$$

where, T_{min} is the minimum natural period in the structure.

Newmark's method is unconditionally stable if satisfying follows:

$$2\beta \geq \gamma \geq \frac{1}{2} \quad (29)$$

However, if γ is greater than $1/2$, it is known that errors are introduced. Therefore, the value of $\gamma = 1/2$ is normally used.

■Average acceleration method The average acceleration method is unconditionally stable because of $\gamma = 1/2$, $\beta = 1/4$.

■Linear acceleration method The linear acceleration method is conditionally stable because of $\gamma = 1/2$, $\beta = 1/6$. The condition of stability is shown below:

$$\frac{\Delta t}{T_{min}} \leq \frac{1}{2\pi\sqrt{\gamma/2 - \beta}} = 0.5513 \quad (30)$$

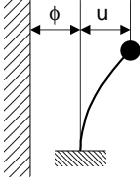
If we use the time interval $\Delta t = 0.01(\text{sec})$, the condition of stability is $T_{min} \geq 0.018(\text{sec})$. In the case of calculation of response spectrum, the time interval of observed earthquake record is usually $0.01(\text{sec})$ and the calculation is started from the natural period of $0.02(\text{sec})$. Therefore, the condition of stability is satisfied.

However, in the case of multi-degree of freedom model such as FEM model, the minimum natural period may become less than $0.018(\text{sec})$. Therefore, the use of the average acceleration method is better than the use of the linear acceleration method in the case of the calculation for multi-degree of freedom model.

2. Response Spectrum

2.1 Acceleration response spectrum

The equation of motion of the point mass on the ground is shown below. In this case, the acceleration of the ground acts to the point mass.



$$m \cdot (\ddot{\phi} + \ddot{u}) + c \cdot \dot{u} + k \cdot u = 0 \quad (31)$$

In above, the acceleration of the point mass is equal to the total of the ground acceleration $\ddot{\phi}$ and the acceleration \ddot{u} which is measured from the reference point in the structure. The value $\ddot{\phi} + \ddot{u}$ is called the absolute acceleration.

On the other hand, the damping force and the restoring force are expressed using \dot{u} and u which are measured from the reference point in the structure. The values \dot{u} and u are called relative velocity and relative displacement, respectively.

The response spectrum is used for the understanding the characteristics of a earthquake motion. And the response spectrum is expressed as a graph which indicates the relationship between the natural frequency of SDOF model and the maximum value of the absolute acceleration or the relative velocity or the relative displacement.

Usually, following formula is used for the calculation of the response spectrum. This can be obtained by dividing through by m of original equation of motion and transposition of the ground acceleration $\ddot{\phi}$.

$$\ddot{u} + 2 \cdot h \cdot \omega_0 \cdot \dot{u} + \omega_0^2 \cdot u = -\ddot{\phi} \quad (32)$$

where, h : damping factor of the system, ω_0 : natural circular frequency of the system.

The relationships between h , ω_0 and m , c , k are shown below:

$$c = 2 \cdot h \sqrt{k \cdot m} \quad \omega_0 = \sqrt{\frac{k}{m}} \quad T = \frac{2\pi}{\omega_0} = 2\pi \sqrt{\frac{m}{k}} \quad (33)$$

where, T is the natural period of the system.

In this section, we consider the program for calculation of the acceleration response spectrum for the earthquake motion.

As a format of the equation of motion, we use following format taking into consideration of the use of dynamic analysis of a general structure. Input data are the ground acceleration by a earthquake $\ddot{\phi}$, the damping factor h and the natural period of system T .

$$m \cdot \ddot{u} + c \cdot \dot{u} + k \cdot u = -m \cdot \ddot{\phi} \quad (34)$$

$$m = 1 \quad k = \frac{4\pi^2 \cdot m}{T^2} \quad c = 2 \cdot h \sqrt{k \cdot m} \quad (35)$$

The basic setting in the program is shown below:

Time interval of the ground acceleration record	: $\Delta t(\text{sec})$
Calculation range of the natural period	: $T = 2 \cdot \Delta t \sim 10(\text{sec})$
Damping factor	: $h = 0.05$

Note that the maximum value of the absolute acceleration $\ddot{u} + \ddot{\phi}$ shall be searched and outputted as a acceleration response. That is, since the relative acceleration \ddot{u} is obtained as a solution of the equation of motion by the numerical integration, it is necessary to get the absolute acceleration by adding the ground acceleration $\ddot{\phi}$ to the relative acceleration \ddot{u} .

In used program, the equation of motion is solved using the linear acceleration method. And the base-line correction of accelerogram and the integration of acceleration time history for base-line correction are carried out using the subroutines by Dr.Ohsaki. (Although the original subroutines are written using FORTRAN 77, these are rewritten into Fortran 90.)

The source data for sample calculations were downloaded from 'KiK-net' of National Research Institute for Earth Science and Disaster Prevention in Japan.

The time history data of ground acceleration inputted is shown in **Fig.1**, the acceleration response spectrum by the linear acceleration method is shown **Fig.2**. Red lines are by Dr.Ohsaki's subroutine 'ERES'. Both lines are almost the same.

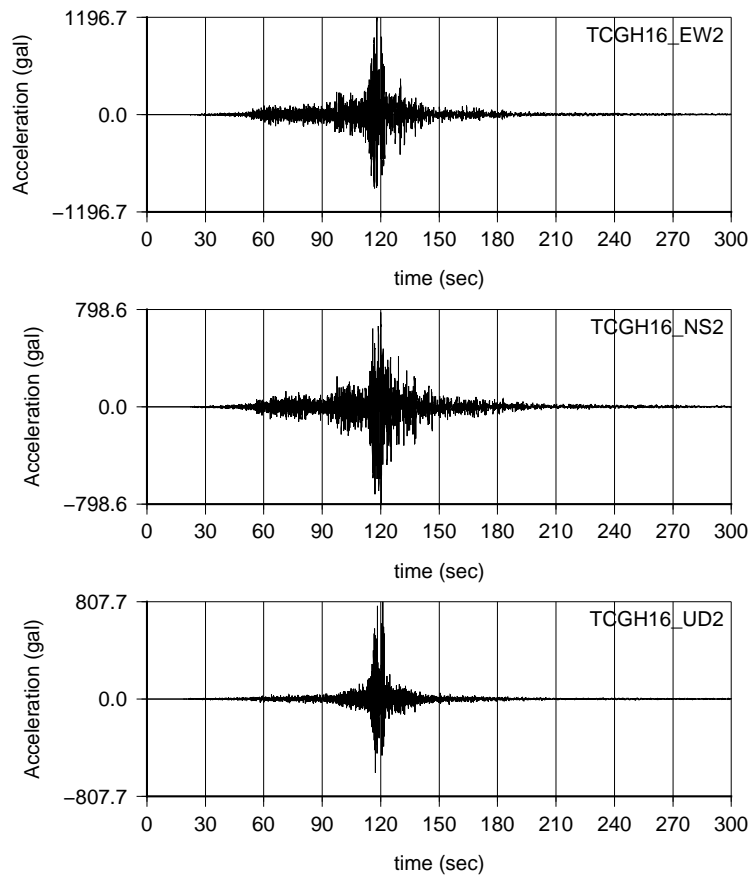


Fig.1 Time history data of the ground acceleration inputted

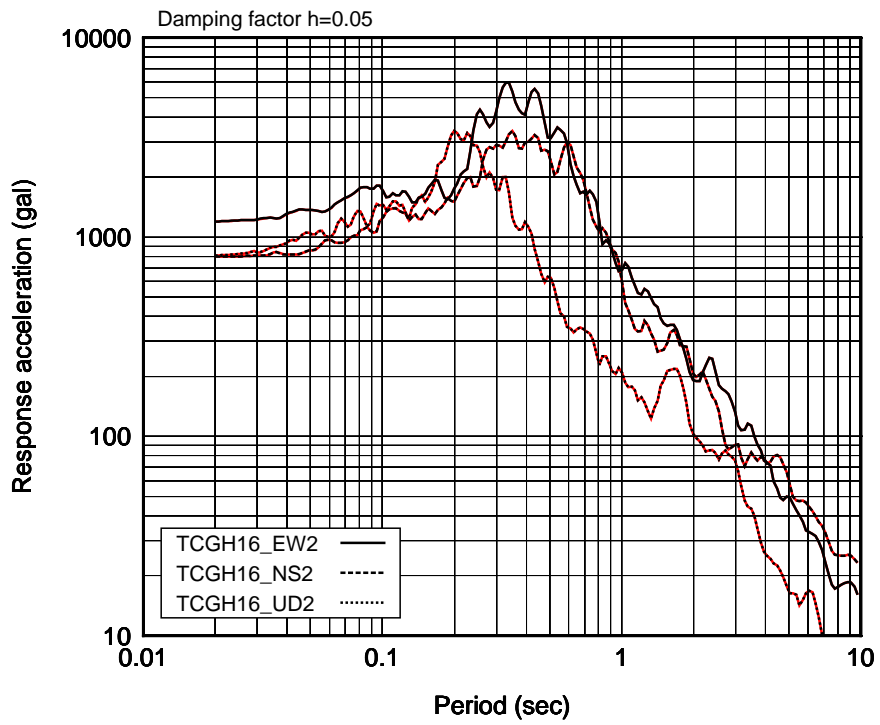


Fig.2 Acceleration response spectrum by the linear acceleration method

2.2 Tripartite response spectrum

In the graph of the tripartite response spectrum, the velocity response spectrum is drawn on the Log-Log plane firstly, next axes for acceleration and displacement responses are appended. By using this one, we can know the approximate values of the acceleration, velocity and displacement from only one graph.

The axes for acceleration and displacement are defined using following relationship.

$$S_a \doteq \frac{2\pi}{T} \cdot S_v \quad S_d \doteq \frac{T}{2\pi} \cdot S_v \quad (36)$$

where, S_a : Maximum absolute acceleration response
 S_v : Maximum relative velocity response
 S_d : maximum relative displacement response

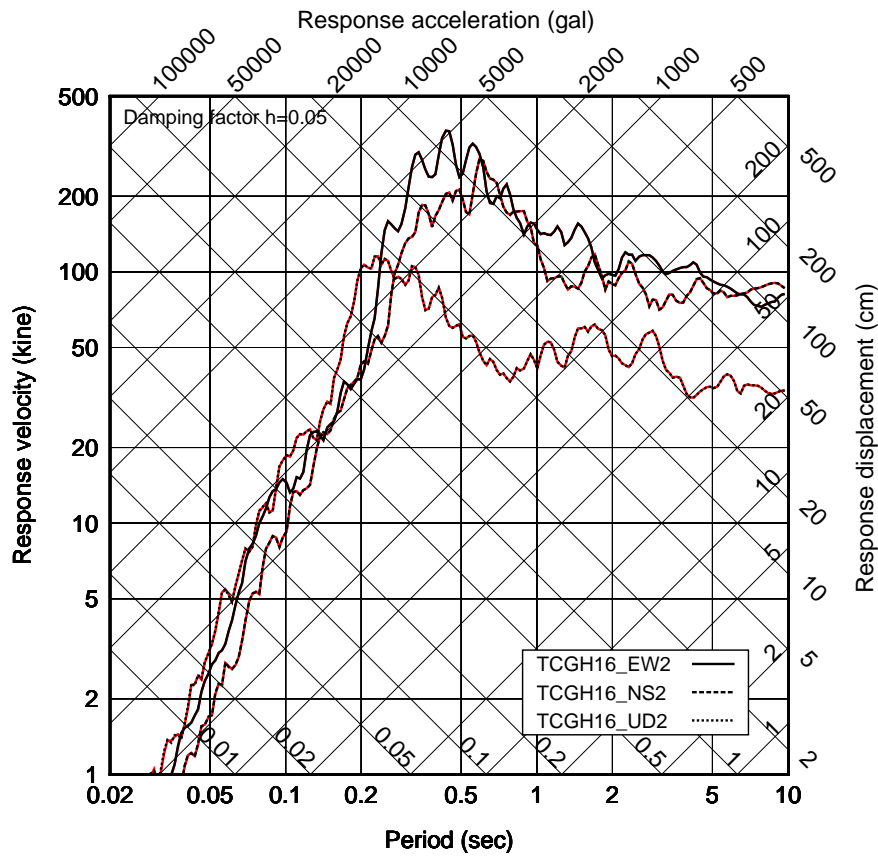


Fig.3 Sample of the tripartite response spectrum

3. Fourier spectrum

3.1 Theory

(1) Finite Fourier coefficient

Consider the following digital data.

Sample number	N	
Time interval	Δt	
Digital value	x_m	$m = 0, 1, 2, \dots, N - 1$

Set the sample number $N = 2^L$ (L : positive integer) as a velu of a power of two in order to adopt the FFT. And if we deem that x_m can be expressed using the finite trigonometrical from 0 to $N/2$, following form can be obtained.

$$\begin{aligned}
 x_m &= \sum_{k=0}^{N/2} \left\{ A_k \cdot \cos \left(\frac{2\pi k}{N \cdot \Delta t} \cdot t \right) + B_k \cdot \sin \left(\frac{2\pi k}{N \cdot \Delta t} \cdot t \right) \right\} \\
 &= \frac{A_0}{2} + \sum_{k=1}^{N/2-1} \left(A_k \cdot \cos \frac{2\pi km}{N} + B_k \cdot \sin \frac{2\pi km}{N} \right) + \frac{A_{n/2}}{2} \cdot \cos \frac{2\pi(N/2)m}{N} \\
 & \quad m = 0, 1, 2, \dots, N - 1 \quad t = m \cdot \Delta t
 \end{aligned}$$

Above form is called the finite Fourier approximation of the function $x(t)$, and A_k, B_k are called finite Fourier coefficients.

The mathematical operation to obtain the finite Fourier coefficients is called Fourier transform.

The mathematical operation to obtain the digital sample data using known finite Fourier coefficients is called inverse Fourier transform.

$$A_k = \frac{2}{N} \sum_{m=0}^{N-1} x_m \cdot \cos \frac{2\pi km}{N} \quad k = 0, 1, 2, \dots, N/2 - 1, N/2 \quad (37)$$

$$B_k = \frac{2}{N} \sum_{m=0}^{N-1} x_m \cdot \sin \frac{2\pi km}{N} \quad k = 1, 2, \dots, N/2 - 1 \quad (38)$$

$$\frac{A_0}{2} = \frac{1}{N} \sum_{m=0}^{N-1} x_m \quad (\text{Average of all sample data}) \quad (39)$$

(2) Expression of finite Fourier transform using complex number

Express the Fourier transform and inverse Fourier transform in complex number using Euler's formula ($e^{\pm i\theta} = \cos \theta \pm i \sin \theta$).

$$\text{Fourier transform} \quad C_k = \frac{1}{N} \sum_{m=0}^{N-1} x_m \cdot e^{-i(2\pi km/N)} \quad k = 0, 1, 2, \dots, N - 1 \quad (40)$$

$$\text{Inverse Fourier transform} \quad x_m = \sum_{k=0}^{N-1} C_k \cdot e^{+i(2\pi km/N)} \quad m = 0, 1, 2, \dots, N - 1 \quad (41)$$

(3) Relationship between real Fourier coefficient and complex Fourier coefficient

$$\begin{aligned}
 C_k &= \frac{A_k - iB_k}{2} & A_k + iB_k &= A_{N-k} - iB_{N-k} \\
 A_k &= 2 \cdot \text{Re}(C_k) & B_k &= -2 \cdot \text{Im}(C_k) \quad k = 0, 1, 2, \dots, N/2
 \end{aligned}$$

(4) Fourier amplitude spectrum and Fourier phase spectrum

Frequency	$f_k = \frac{k}{N \cdot \Delta t} \quad k = 0, 1, 2, \dots, N/2$
Fourier amplitude	$F_k = \frac{N \cdot \Delta t}{2} \sqrt{A_k^2 + B_k^2} = N \cdot \Delta t C_k = N \cdot \Delta t \sqrt{Re(C_k)^2 + Im(C_k)^2}$
Phase angle	$\phi_k = \arctan\left(-\frac{B_k}{A_k}\right) = \arctan\left(\frac{Im(C_k)}{Re(C_k)}\right) \quad (-\pi < \phi_k < \pi)$
Phase wave	$\hat{x}(t) = \sum_{k=1}^{N/2} \cos(2\pi \cdot f_k \cdot t + \phi_k)$

The plot which shows the relationship between frequencies and Fourier amplitudes are called Fourier amplitude spectrum. This can be called only Fourier spectrum.

The plot which shows the relationship between frequencies and phase angles are called Fourier phase spectrum.

3.2 Simple use of Fourier transform and inverse Fourier transform

The results of use of Fourier transform and inverse Fourier transform are shown below:

- Data was made using Fortran 90 subroutine 'random_number' with a sample number of 16.
- Re(Ck) and Im(Ck) are complex Fourier coefficient by Fast Fourier Transform.
- abs(Ck) is absolute value of complex Fourier coefficient, and fp(k) is phase angle.
- Re(xm) and Im(xm) are re-produced value using complex Fourier coefficient by inverse Fourier transform.

i	data	Re(Ck)	Im(Ck)	abs(Ck)	fp(Ck)	Re(xm)	Im(xm)
0	0.998	0.478	0.000	0.478	0.000	0.998	0.000
1	0.567	0.154	-0.014	0.154	-5.171	0.567	0.000
2	0.966	-0.003	-0.053	0.053	-93.070	0.966	0.000
3	0.748	-0.018	-0.008	0.020	-155.386	0.748	0.000
4	0.367	0.057	-0.014	0.059	-14.125	0.367	0.000
5	0.481	0.000	0.092	0.092	89.861	0.481	0.000
6	0.074	0.012	0.030	0.033	67.645	0.074	0.000
7	0.005	0.027	-0.047	0.054	-60.520	0.005	0.000
8	0.347	0.062	0.000	0.062	0.000	0.347	0.000
9	0.342	0.027	0.047	0.054	60.520	0.342	0.000
10	0.218	0.012	-0.030	0.033	-67.645	0.218	0.000
11	0.133	0.000	-0.092	0.092	-89.861	0.133	0.000
12	0.901	0.057	0.014	0.059	14.125	0.901	0.000
13	0.387	-0.018	0.008	0.020	155.386	0.387	0.000
14	0.445	-0.003	0.053	0.053	93.070	0.445	0.000
15	0.662	0.154	0.014	0.154	5.171	0.662	0.000
Ave.	0.478						

From above table, we can know follows and they are expected results.

- Regarding the real parts of complex Fourier coefficients Re(Ck), 1st value for i=0 is equal to the average of all real data. And distribution of Re(Ck) is line symmetry to the point of $n/2 + 1$ th (i=8), where n means sample number.
- Regarding the imaginary parts of complex Fourier coefficients Im(Ck), 1st value for i=0 is equal to zero. And distribution of Im(Ck) is point symmetry to the point of $n/2 + 1$ th (i=8), where n means sample number.
- Regarding the result of inverse Fourier transform, the values of real parts Re(xm) are corresponded to inputted data, and all values of imaginary parts Im(xm) are deemed approximately zero.

In this section, FFT program by Fortran 90 is used. This program was rewritten into Fortran 90 from BASIC program by Dr.Minami.

Note following items.

- ① For Fourier transform Since this program outputs the parts of $\sum_{m=0}^{N-1} x_m \cdot e^{-i(2\pi km/N)}$ in Equation(40),

outputted data shall be divided by the sample number N .

- ②For inverse Fourier transform In order to obtain the value of acceleration from given complex Fourier coefficients by inverse Fourier transform, the sign of imaginary parts of input data shall be reversed. We can understand the necessity of this treatment by comparison of the exponential parts between Equation(40) and Equation(41).

The program for FFT and IFFT by Fortran 90 is shown below.

```

module defpi
  implicit none
  real(8), parameter :: pi=3.14159265358979323846D0
end module defpi

program f90_FFT
  !*****
  ! FFT and Inverse FFT
  !*****
  use defpi
  implicit none
  integer :: i
  integer :: ndata
  integer :: nn
  real(8) :: dt
  real(8), allocatable :: accinp(:)
  real(8), allocatable :: xr(:)
  real(8), allocatable :: xi(:)
  real(8), allocatable :: cr(:)
  real(8), allocatable :: ci(:)
  real(8), allocatable :: ck(:)
  real(8), allocatable :: fp(:)
  character :: fnameR*50, fnameW*50
  character :: strcom*200, dummy*50, fmt1*200
  character :: linebuff*1000

  call getarg(1, fnameR) !Input data file name
  call getarg(2, fnameW) !Input output file name

  open(11, file=fnameR, status='old')
  read(11, '(a)') strcom
  read(11, *) dummy, dt
  read(11, *) dummy, ndata
  !To set number of data
  ! (nn is equal to 2 to the power of positive integer number
  ! and nn is greater than or equal to number of data)
  nn=2
  do
    nn=nn*2
    if(ndata*1<=nn) exit
  end do
  allocate(accinp(1:nn))
  allocate(xr(1:nn))
  allocate(xi(1:nn))
  allocate(cr(1:nn))
  allocate(ci(1:nn))
  allocate(ck(1:nn))
  allocate(fp(1:nn))
  do i=1,nn
    accinp(i)=0.0D0
    xr(i)=0.0D0
    xi(i)=0.0D0
  end do
  do i=1,ndata
    read(11, *) accinp(i)
    xr(i)=accinp(i)
  end do
  close(11)

  call FFT(nn, xr, xi) !Fourier transform
  do i=1,nn !To divide returned values by number of data
    xr(i)=xr(i)/dble(nn)
    xi(i)=xi(i)/dble(nn)
    cr(i)=xr(i) !To memory real part of transformed values
    ci(i)=xi(i) !To memory imaginal part of transformed values
  end do

```

```

ck(i)=sqrt(cr(i)**2.0D0+ci(i)**2.0D0)
fp(i)=0.0D0
if(1.0D-30<abs(cr(i)))then
    fp(i)=atan(ci(i)/cr(i))/pi*180.0D0
    if(cr(i)<0.0D0.and.ci(i)<0.0D0)fp(i)=fp(i)-180.0D0
    if(cr(i)<0.0D0.and.0.0D0<ci(i))fp(i)=180.0D0+fp(i)
else
    if(0.0D0<ci(i).and.0.0D0<cr(i))fp(i)=90.0D0
    if(ci(i)<0.0D0.and.cr(i)<0.0D0)fp(i)=-90.0D0
    if(0.0D0<ci(i).and.cr(i)<0.0D0)fp(i)=90.0D0
    if(ci(i)<0.0D0.and.0.0D0<cr(i))fp(i)=-90.0D0
end if
end do
do i=1,nn !To change the sign of imaginary parts for inverse FFT
    xr(i)=xr(i)
    xi(i)=-xi(i)
end do
Call FFT(nn,xr,xi) !Inverse Fourier transform

fmt1="(i5,' ',e15.7,4(' ',e15.7),2(' ',e15.7))"
open(12,file=fnameW,status='replace')
write(12,'(a)') 'i,data,Re(Ck),Im(Ck),abs(Ck),fp(Ck),Re(xm),Im(xm)'
do i=1,nn
    write(linebuff,FMT=fmt1) i,accinp(i),cr(i),ci(i),ck(i),fp(i),xr(i),xi(i)
    call del_spaces(linebuff)
    write(12,'(a)') trim(linebuff)
end do
close(12)

```

stop

contains

```

subroutine FFT(nn,xr,xi)
!*****
!Fast Fourier Transform, Inverse transform
!*****
!nn : Number of data (powers of 2)
!xr() : Real part of i/o data
!xi() : Imaginary part of i/o data
integer,intent(in):: nn
real(8),intent(inout):: xr(1:nn)
real(8),intent(inout):: xi(1:nn)

integer:: g,h,i,j,k
integer:: l,m,n,p,q
real(8):: a,b,xd
real(8),allocatable:: s(:)
real(8),allocatable:: c(:)

n=nn
allocate(s(1:n/2+1))
allocate(c(1:n/2+1))
i=0;j=0;k=0;l=0;p=0;h=0;g=0;q=0
m=int(log(dble(n))/log(2.0D0)+1.0D0)
a=0.0D0
b=pi*2.0D0/dble(n)
do i=0,n/2
    s(i+1)=sin(a)
    c(i+1)=cos(a)
    a=a+b
end do
l=n
h=1
do g=1,m
    l=l/2
    k=0
    do q=1,h
        p=0
        do i=k,l+k-1
            j=i+1
            a=xr(i+1)-xr(j+1)
            b=xi(i+1)-xi(j+1)
            xr(i+1)=xr(i+1)+xr(j+1)
            xi(i+1)=xi(i+1)+xi(j+1)
            if(p==0)then

```

```

                xr(j+1)=a
                xi(j+1)=b
            else
                xr(j+1)=a*c(p+1)+b*s(p+1)
                xi(j+1)=b*c(p+1)-a*s(p+1)
            end if
            p=p+h
        end do
        k=k+1+1
    end do
    h=h+h
end do
j=n/2
do i=1,n-1
    k=n
    if(j<i)then
        xd=xr(i+1)
        xr(i+1)=xr(j+1)
        xr(j+1)=xd
        xd=xi(i+1)
        xi(i+1)=xi(j+1)
        xi(j+1)=xd
    end if
    k=k/2
    do while(j>=k)
        j=j-k
        k=k/2
        if(k==0)exit
    end do
    j=j+k
end do
end subroutine FFT

```

```

subroutine del_spaces(s)
    character (*), intent (inout) :: s
    character (len=len(s)) tmp
    integer i, j
    j = 1
    do i = 1, len(s)
        if (s(i:i)==' ') cycle
        tmp(j:j) = s(i:i)
        j = j + 1
    end do
    s = tmp(1:j-1)
end subroutine del_spaces

```

end program f90_FFT

3.3 Sample drawing of Fourier spectrum

Following relationship is drawn in the drawing of Fourier spectrum.

Frequency	$f_k = \frac{k}{N \cdot \Delta t}$	$(k = 0 \sim N/2)$
Fourier amplitude	$F_k = N \cdot \Delta t \sqrt{Re(C_k)^2 + Im(C_k)^2}$	$(k = 0 \sim N/2)$

Fourier spectra for the ground accelerations shown in **Fig.1** are indicated in **Fig.4**. In the drawing **Fig.4**, black line means smoothed line by Parzen window with band width of 1.0Hz, and red line means non-smoothed results. Used subroutine for Parzen window is rewritten into Fortran 90 from original subroutine 'SWIN' by Dr.Ohsaki.

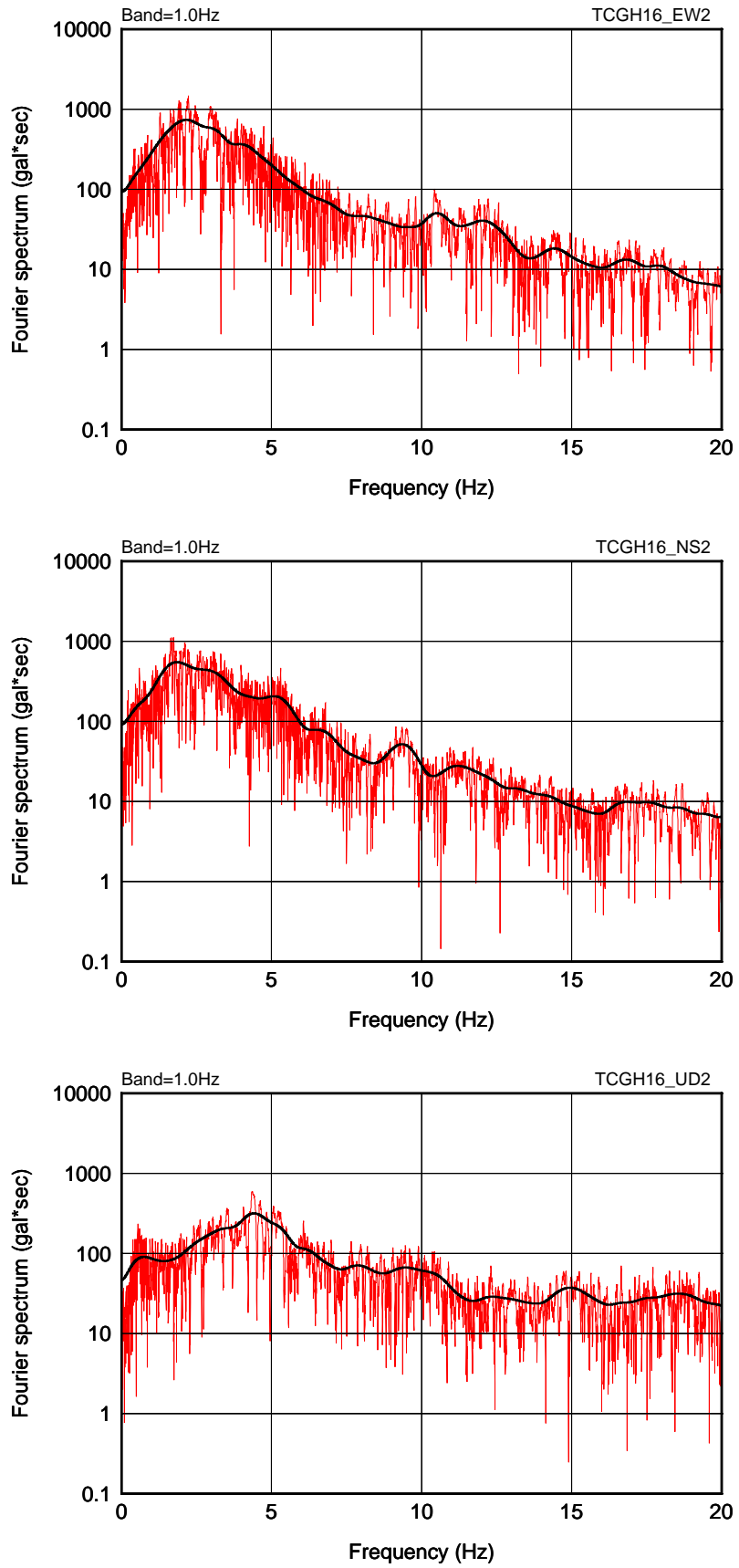


Fig.4 Sample drawings of Fourier spectra
 (Black line: smoothed, Red line: non-smoothed)

4. Simulated earthquake motion

4.1 Basic concept for making a simulated earthquake motion

- Basic concept of this method is referred to 'Technical Note of national Institute for Land and Infrastructure Management, No.244, march 2005 (Guideline for the Seismic Performance Evaluation of dams against large Earthquake (Draft))'.
- The target acceleration spectrum and initial wave (acceleration time history data) are required as input, in this method, and output is the acceleration time history data which fits the specified target spectrum.
- Allowable error for fitting is 5%. However, in case that error doesn't achieve the allowable value and error reaches the bottom, the iterative process is stopped.
- The time range of simulated earthquake motion is the same as it of initial wave inputted.
- Although the target spectrum is inputted as a function of the period, it is treated as a function of the frequency to adopt Fourier transform in the internal processing.

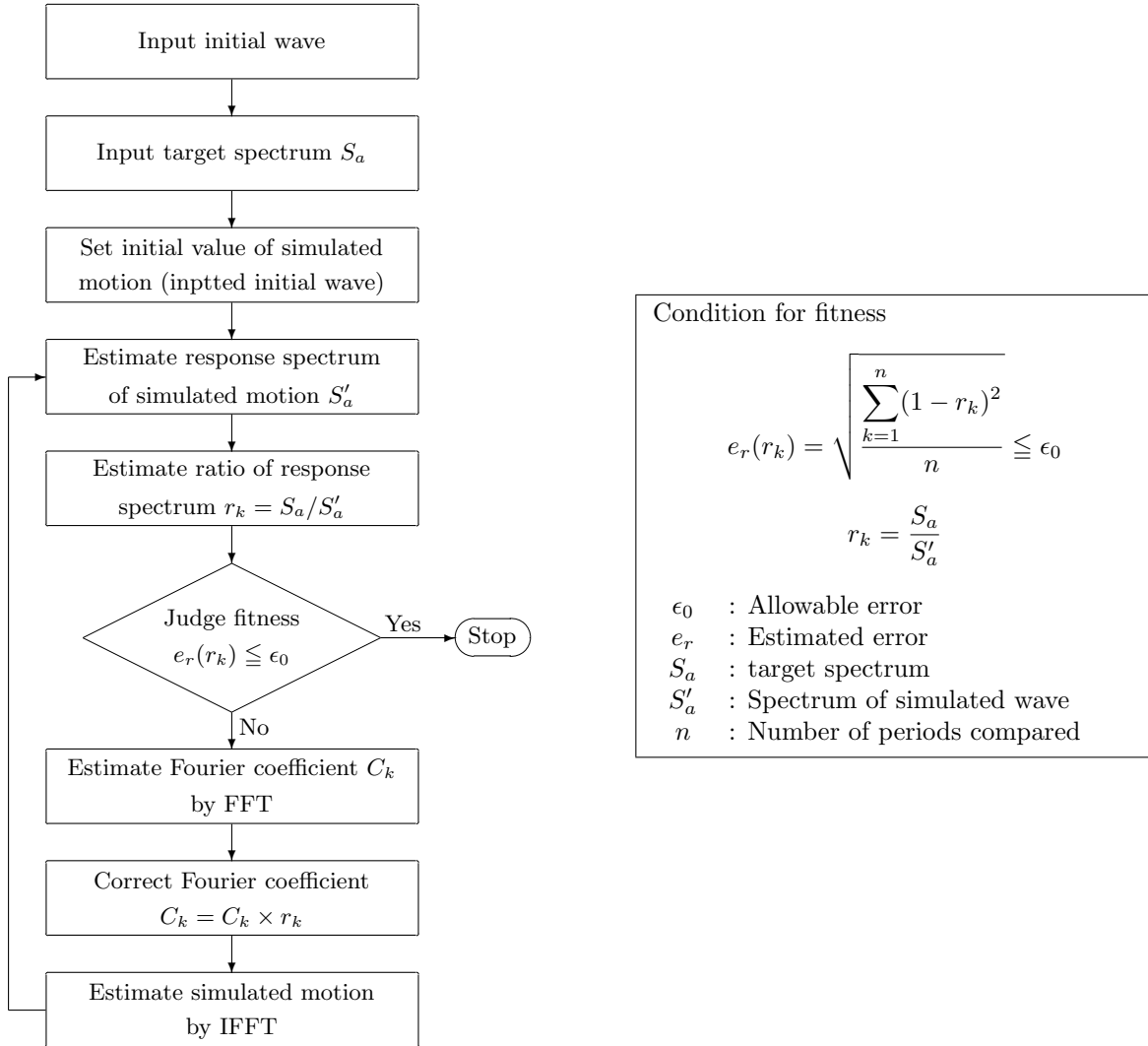


Fig.5 Process for making a simulated earthquake motion

4.2 Effect of duration time of earthquake

Try to know the effect of duration time of earthquake using mentioned process. A Fortran program was made for this trial.

Duration time of earthquake was estimated using following thinking.

Case considered				Envelope function	
Case	$a(\text{sec})$	$b(\text{sec})$	$c(\text{sec})$	$\begin{cases} e(t) = (t/a)^2 \\ e(t) = 1.0 \\ e(t) = \exp\{-\alpha \cdot (t - b)\} \\ e(t) = 0.1 \quad \text{at } t = c \end{cases} \quad \alpha = -\frac{\ln(0.1)}{c - b}$	
1	5	15	30		
2	5	25	60		
3	5	35	120		

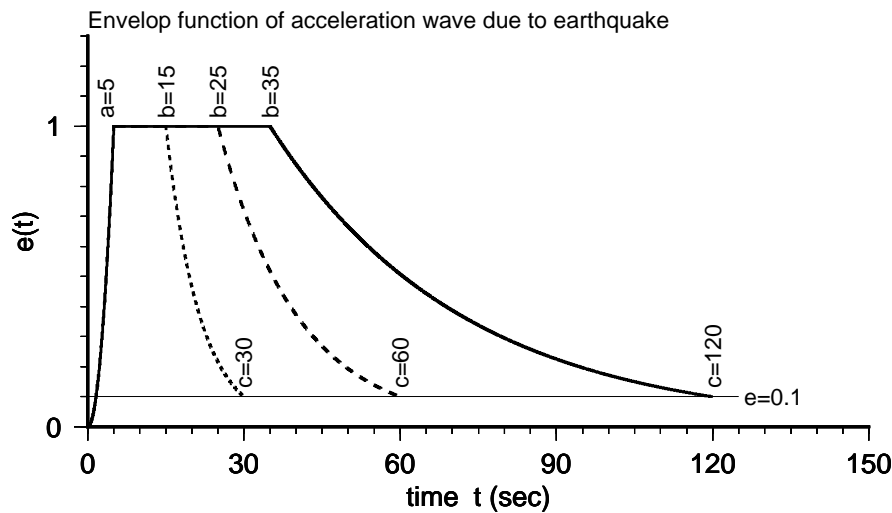


Fig.6 Envelope function

Initial waves for this trial were made by following process.

- Obtain dataset of uniform random numbers with the range of $0 \sim 1$ using the subroutine 'random_number' in Fortran 90.
- Enlarge the range of dataset to $-1 \sim 1$, and obtain the initial wave by multiplying enlarged dataset of random numbers by envelope function and specified maximum acceleration.
- Time interval is set to $\Delta t = 0.01(\text{sec})$.
- maximum acceleration is set to 100 gal.

Some simulated earthquake motions were made using initial waves which were produced by above process. Maximum acceleration of target spectrum was set to 300 gal.

After this, initial wave, simulated earthquake motion and acceleration response spectrum for each case are shown. Please note that the purpose of these trial is only numerical testing, not practical work.

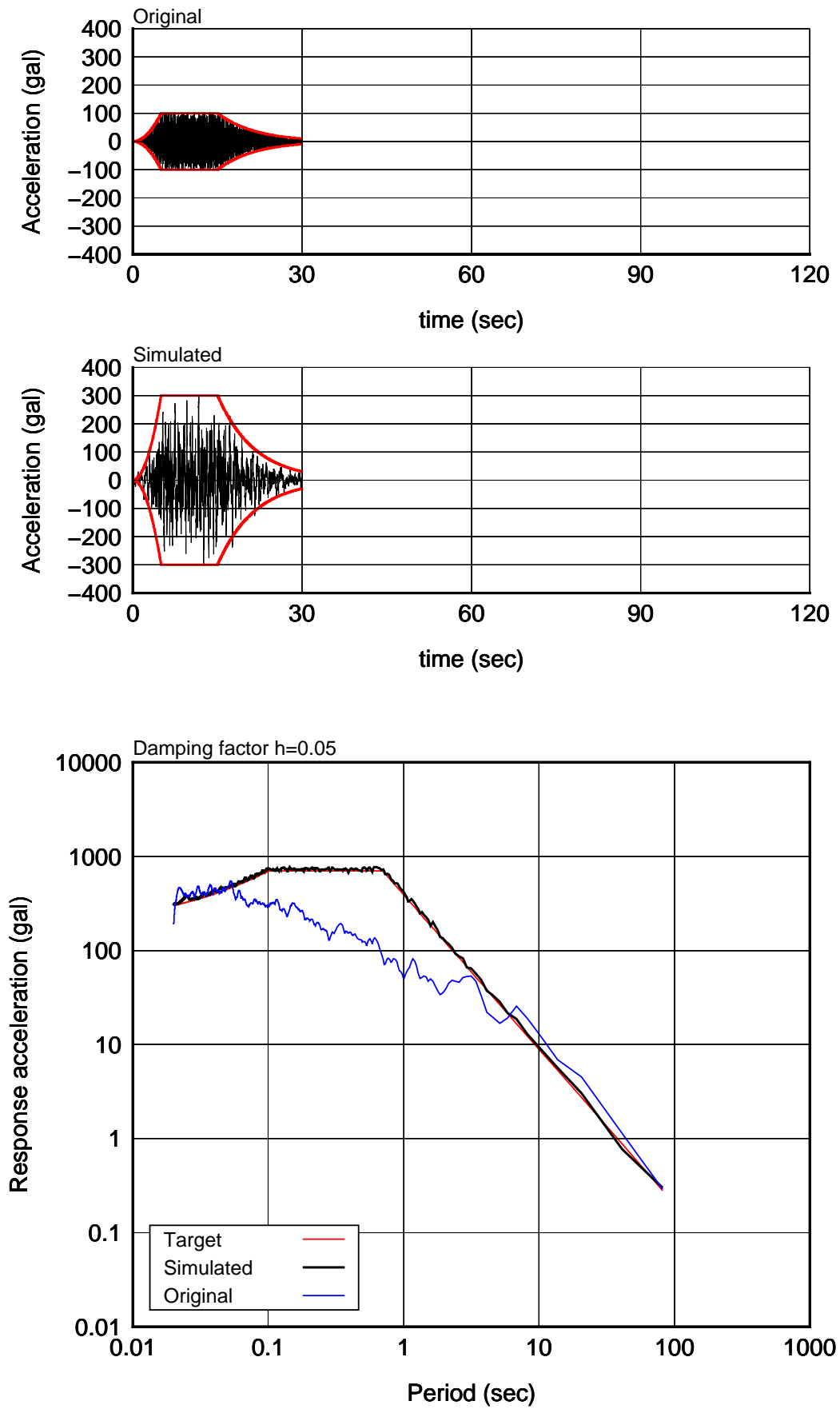


Fig.7 Case 1 (duration time: C=30 sec)

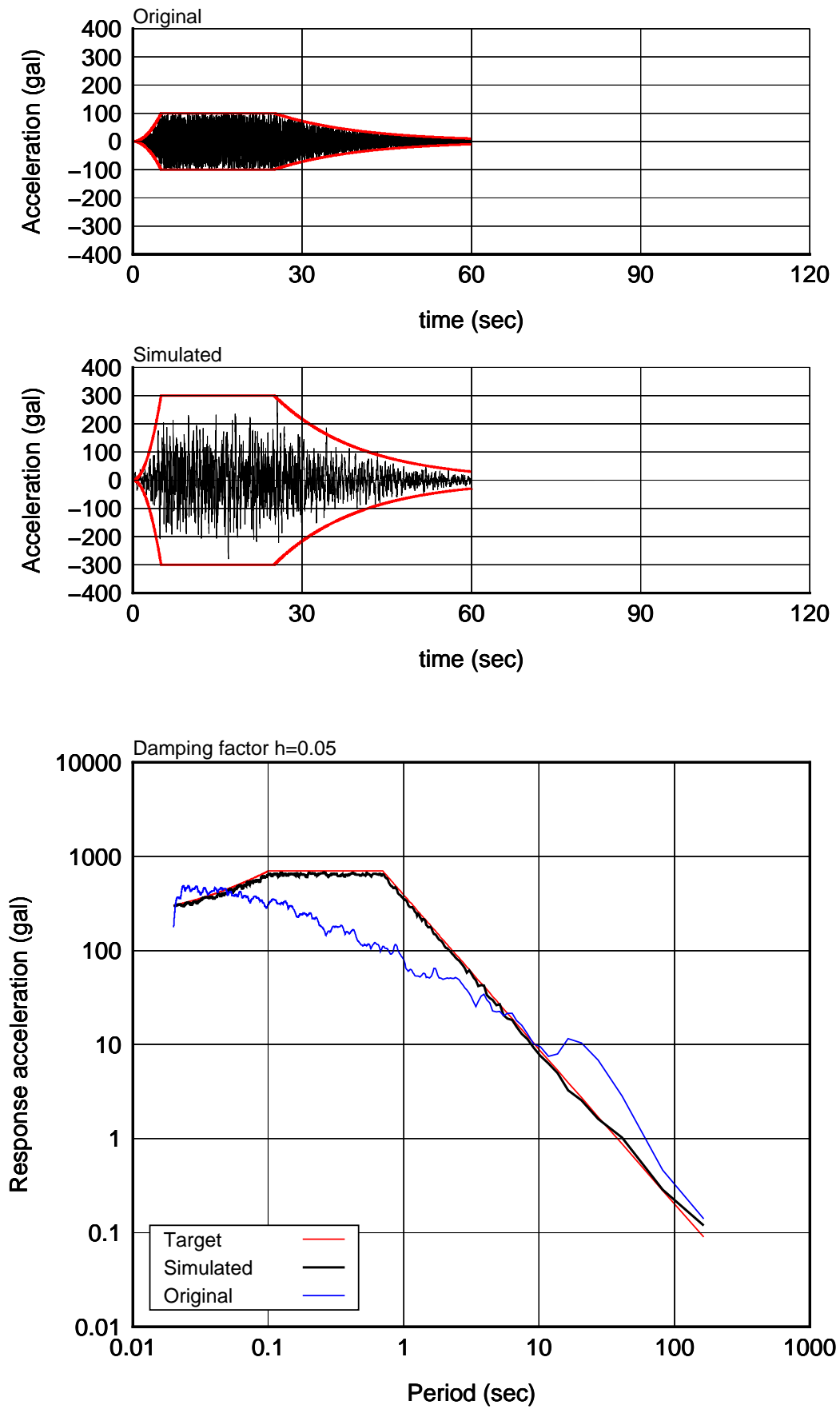


Fig.8 Case 2 (duration time: C=60 sec)

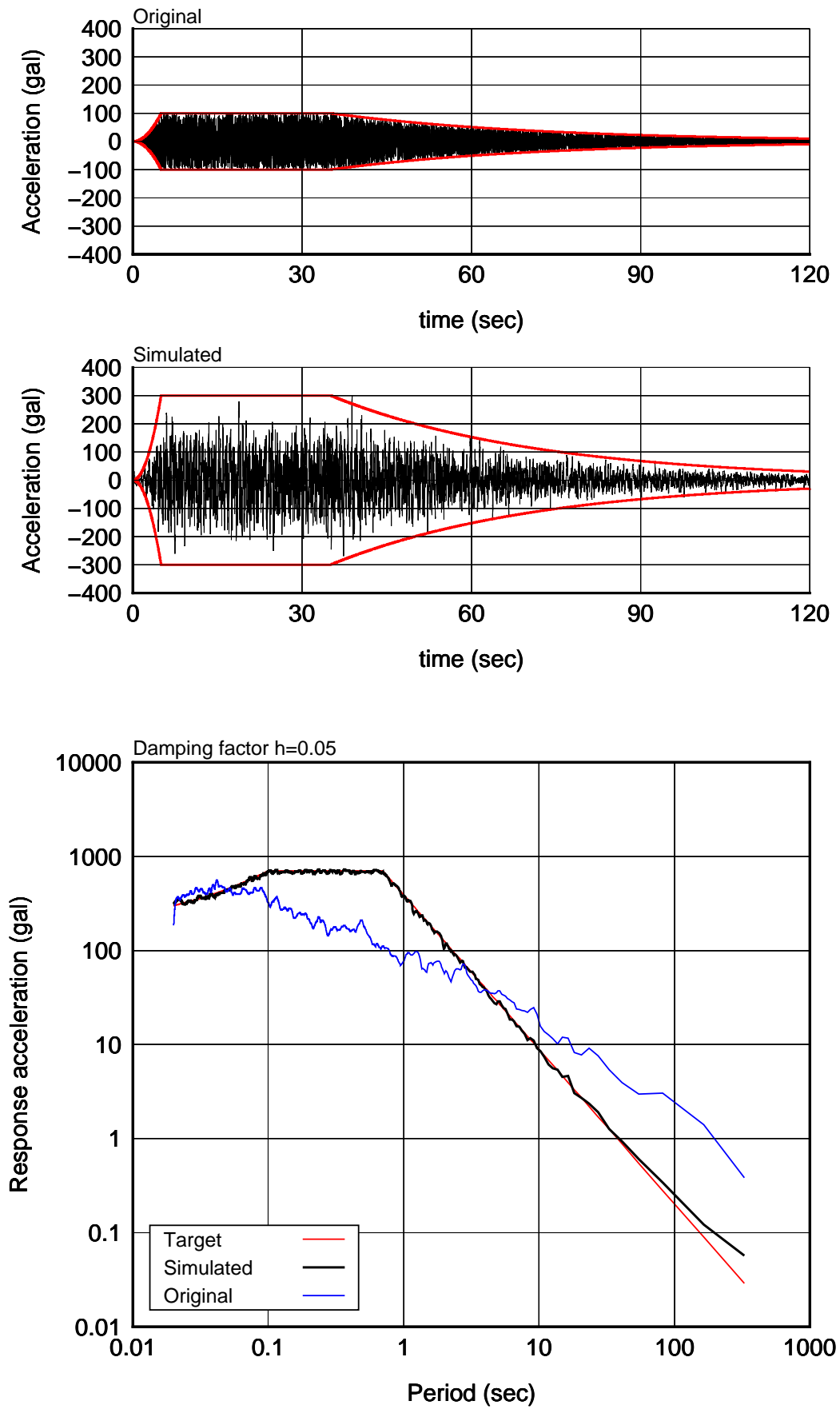


Fig.9 Case 3 (duration time: C=120 sec)

4.3 Phase angle (sample using Case 1)

- The phase angles of initial wave are quite same as them of simulated earthquake motion.
- Above fact means that the phase angles of initial wave are maintained and Fourier amplitudes are changed to fit the target spectrum in used process.
- The shape of phase wave made from simulated earthquake motion is almost like it made from initial wave.
- Although the reason above can not be understood, it seems that the shape of phase wave of initial wave affects duration time and the shape of envelope function.

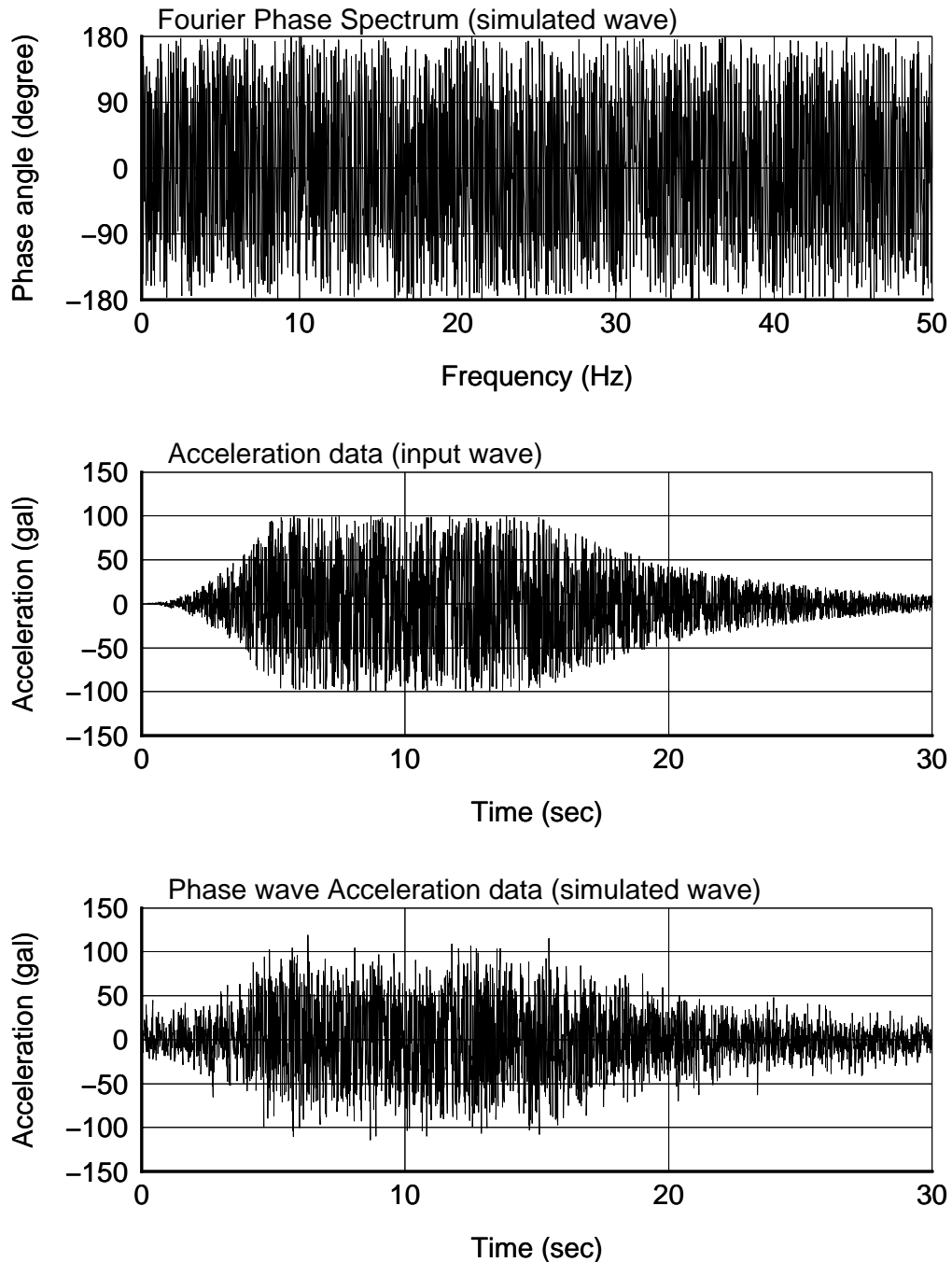


Fig.10 Phase spectrum and phase wave (Case 1)

5. Frequency dmain analysis for SDOF system

As you know, there are two typical methods for numerical integration, one is a time domain analysis, and the ather is a frequency dmain analysis.

In this section, frequency dmain analysis for SDOF system is introduced.

The equation of motion is shown below: .

$$\ddot{u}(t) + 2 \cdot h \cdot \omega_0 \cdot \dot{u}(t) + \omega_0^2 \cdot u(t) = -a(t) \quad (42)$$

where, u : relative displacement of point mass, a : acceleration of the ground, h : damping factor of system, ω_0 : circular natural frequency of system.

The acceleration of the ground, the displacement, velocity, acceleration of point mass are expressed using inverse Fourier transform as follow:

$$a(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} A(\omega) e^{i\omega t} d\omega \quad (43)$$

$$u(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(\omega) e^{i\omega t} d\omega \quad (44)$$

$$\dot{u}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} i\omega \cdot U(\omega) e^{i\omega t} d\omega \quad (45)$$

$$\ddot{u}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} -\omega^2 \cdot U(\omega) e^{i\omega t} d\omega \quad (46)$$

Substituting Equation(43)(44)(45)(46) to Equation(42), responses of displacement, velocity and acceleration can be obtain as follows:

$$U(\omega) = \frac{A(\omega)}{(\omega^2 - \omega_0^2) - 2h\omega_0\omega \cdot i} \quad (47)$$

$$u(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{A(\omega)}{(\omega^2 - \omega_0^2) - 2h\omega_0\omega \cdot i} e^{i\omega t} d\omega \quad (48)$$

$$\dot{u}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{i\omega A(\omega)}{(\omega^2 - \omega_0^2) - 2h\omega_0\omega \cdot i} e^{i\omega t} d\omega \quad (49)$$

$$\ddot{u}(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{-\omega^2 A(\omega)}{(\omega^2 - \omega_0^2) - 2h\omega_0\omega \cdot i} e^{i\omega t} d\omega \quad (50)$$

Where, following function $H(\omega)$ is called transfer funation or frequency response function.

$$H(\omega) = \frac{1}{(\omega^2 - \omega_0^2) - 2h\omega_0\omega \cdot i} \quad (51)$$

The process for calculation is shown below:

- ① Estimate complex Fourier coefficient $A(\omega)$ for inputted acceleration a .
- ② Estimate transfer function $H(\omega)$.
- ③ Estimate time history of displacement $u(t)$ using IFFA for $A(\omega) \cdot H(\omega)$
- ④ Estimate time history of velocity $\dot{u}(t)$ using IFFA for $i\omega \cdot A(\omega) \cdot H(\omega)$
- ⑤ Estimate time history of acceleration $\ddot{u}(t)$ using IFFT for $-\omega^2 \cdot A(\omega) \cdot H(\omega)$

Note following issues because of use of FFT.

- Set number of data N and time interval $Delta t$. Where, N must be a power of two.
- Frequency f_k and circular frequency ω can be obtained as follow:

$$f_k = \frac{k}{N \cdot \Delta t} \quad \omega_k = 2\pi f_k \quad k = 0 \sim N/2 \quad (52)$$

Note that although number of data is N , actual number of frequencies defined is only $N/2 + 1$. So, it is necessary to define the frequencies from $N/2 + 2$ -th to N -th as symmetrical values to $N/2 + 1$ -th value.

- Also in case of calculation for complex Fourier coefficients, the same treatment as above is required, but the sign of imaginary parts shall be reversed.

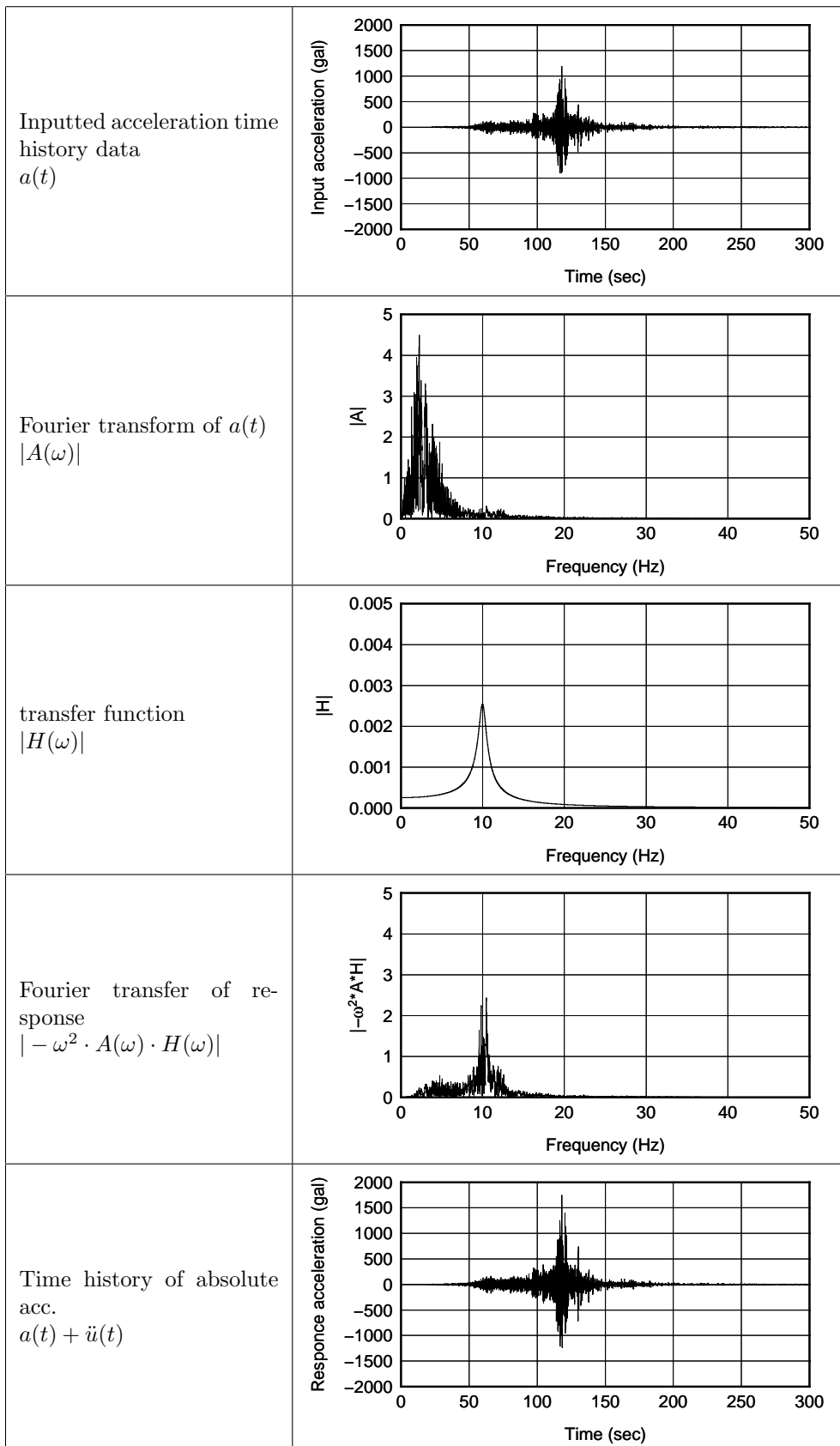


Fig.11 Steps for frequency domain analysis for SDOF (response acceleration)
(damping factor: $h=0.05$, natural frequency: 10 Hz)

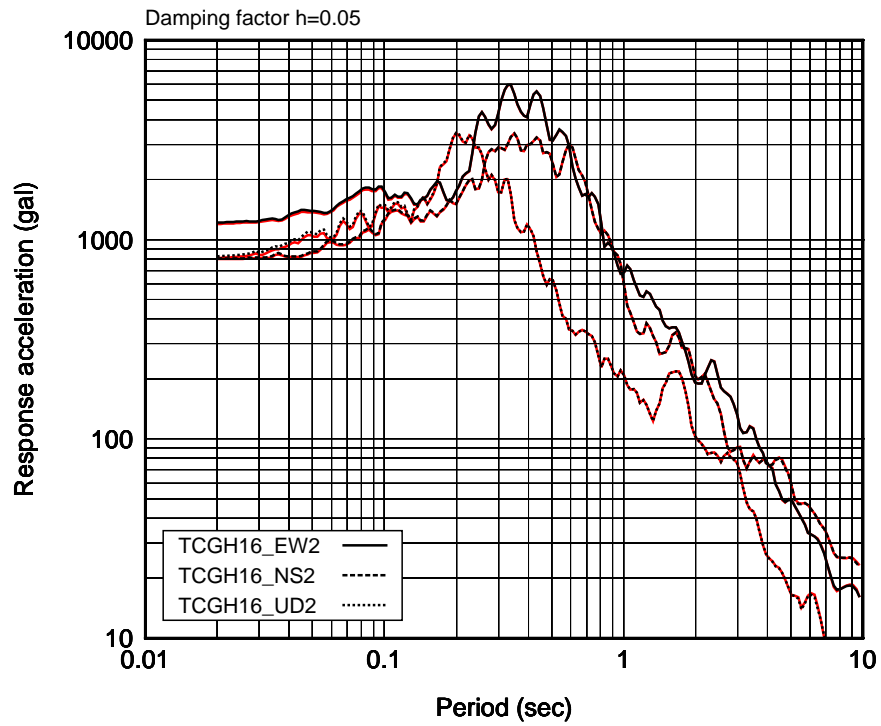


Fig.12 Acceleration response spectrum by frequency domain analysis

The source data for above calculations was downloaded from 'KiK-net' of National Research Institute for Earth Science and Disaster Prevention in Japan. (same data as section 2 was used.)

Red lines are by Dr.Ohsaki's subroutine 'ERES'. Both lines are almost the same.