May 20, 2013

Run-off Analysis using Storage Function Method

1. Equations for analysis

Storage Function Method is framed by following equations.

1.1 Basic equations

$$\frac{dS}{dt} = r_e(t - T_L) - q_c(t) \qquad \text{Equation of continuity} \tag{1}$$

$$S(t) = k \cdot \{q_c(t)\}^p \qquad \text{Relationship between Storage and Direct run-off} \tag{2}$$

$$r_e(t) = f \cdot R(t) \qquad \text{Effective rainfall} \tag{3}$$

$S(t)$	: Storage (mm/hr)	k	: Constant for storage function
$r_e(t)$	: Effective rainfall (mm/hr)	p	: Constant for storage function
T_L	: Delay time (hr)	f	: Rate of flow (constant)
$q_c(t)$	: Direct run-off (mm/hr)	$R(t)$	: Observed rainfall (mm/hr)

1.2 Equations for Direct run-off

$$q_c(t) = q(t) - q_B(t) \qquad \text{Direct run-off} \tag{4}$$

$$q(t) = \frac{3.6 \cdot Q(t)}{A} \qquad \text{Total run-off} \tag{5}$$

$$q_B(t) = q_1 + (i - n_1)\frac{q_2 - q_1}{n_2 - n_1} \qquad \text{Base run-off} \tag{6}$$

$q_c(t)$	: Direct run-off(mm/hr)	$Q(t)$	: Observed discharge (m^3/s)
$q_B(t)$	: Base run-off(mm/hr)	A	: Catchment area (km^2)
$q(t)$	: Total run-off(mm/hr)		
n_1	: Start point of direct run-off	q_1	: Run-off at start point of direct run-off
n_2	: End point of direct run-off	q_2	: Run-off at end point of direct run-off
i	: Integer from n_1 until n_2		

1.3 Equation for the rate of flow

$$f = \frac{\sum q_c}{\sum R_T} \tag{7}$$

f : Rate of flow $\quad q_c$: Direct run-off (mm/hr) $\quad R_T$: Observed rainfall (mm/hr)

where, observed rainfall means that it is shifted one for same times as the times counted direct run-off.

2. Estimation of parameters and reproduction of Hydrograph

Equation of continuity is approximated into following finite difference approximation.

$$\frac{dS}{dt} = r_e(t - T_L) - q_c(t) \quad \Rightarrow \quad S(t + \Delta t) = S(t) + \Delta t \cdot r_e(t + \Delta t - T_L) - \frac{\Delta t}{2}\{q_c(t + \Delta t) + q_c(t)\} \tag{8}$$

In above, since $q_c(t)$ and $r_e(t)$ are known for whole times, the values of $S(t)$ can be obtained successively by setting the initial value of $S(0) = 0$ at $t = 0$.

About delay time T_L, the proper value of it can be obtained by trial changing the value of L_T from zero.

There is following relationship between storage S and direct run-off q_c.

$$S = k \cdot \{q_c\}^p \quad \Rightarrow \quad \ln S = \ln k + p \cdot \ln q_c \tag{9}$$

Practically, by comparison with the relations between the storage S and direct run-off q_c on log-log graph while changing T_L, the value of T_L can be established when an agreement degree of the behavior is the highest during flow increase time and flow decrease time.

The value of k and p can be obtained by a regression analysis on log-log graph using established T_L.

After obtaining the constant values of k, p and T_L, the time history of q_c can be obtained by the numerical integration of following differential equation using such as Runge-Kutta mathod.

$$\frac{dq_c}{dt} = \frac{1}{k \cdot p} \cdot (r_e - q_c) \cdot {q_c}^{1-p} \tag{10}$$

Where, the time history of q_c in above equation is not related the values which were used for obtaining storage S. At this time, the values of r_e, p and k are known and the time history of q_c is unknown. Therefore, it is necessary to set the small initial value of q_c at the equivalent time of starting point of separation n_1. After this, the time history of q_c can be calculated successively using numerical integration.

If the time history of q_c can be known, Hydrograph can be re-producted using following equation.

$$Q = \frac{1}{3.6} \cdot A \cdot (q_c + q_B) \tag{11}$$

Where, please take care of units. The unit of discharge Q is 'm^3/s', the unit for direct run-off q_c and base run-off q_B is 'mm/hr', the unit of catchment area is 'km^2'.

3. Programing

- ◯ The command for execution of Fortran program is shown below:

  ```
  > f90_SFM  fnameR  fnameW  n1  m  md
  ```

  Where, 'fnameR' is a input file name and 'fnameW' is a output file name.
- ◯ Start point of direct run-off n_1 is inputted as a command line argument.
- ◯ About the treatment of the end point of direct run-off n_2, is inputted as a command line argument.
  - In case of $m = 0$, the value of base run-off q_B is constant and the value of it is equal to the value at the start point of direct run-off.
  - In case of $m = 1$, the value of base run-off q_B is variable from the value at n_1 to the value at n_2. The end point of direct run-off n_2 is estimated automatically.
  - The condition of estimation of n_2 is the minimum difference between obsedved from and re-producted flow in Hydrograph.
  - In practical calculation, n_2 is changed from large value to small value.
  - Since the value of direct run-off q_c shall be greater than or equal to zero, the values of direct run-off q_c are amended to zero if they are less than zero.
- ◯ The estimation method of delay time is shown below.
  - Although it is known that the relationship between direct run-off q_c and storage S becomes like a loop shape, the parameters of p and k can be calculated using a regression analysis on log-log graph.
  - By shifting the time for effective rainfall, the delay time T_L can be obtained when the residual error between observed data and estimated data by regression analysis becomes minimum value.
  - For the regression analysis to calculate the constants p and k, special method is adopted to consider the loop shape of q_c-S relationship. The data selection method is shown below:
    - ☐ Divide the area in horizontal axis q_c to some small areas.
    - ☐ Chose the maximum and minimum values of S from each area.
    - ☐ Do the regression analysis using the total datasets which are selected from each small area.
  - The upper limit of value of p is set as 1.0. If the value of p exceed 1.0 is obtained by calculation, p is set by 1.0 and k is re-calculated under the condition of $p = 1.0$.
- ◯ The number of devision of time increment for numerical integration cam be changeable. It is inputted as a number of md of a command line argument. Discharge data is interpolated by Spline interpolation, and rainfall data is set as same value during one hour because the unit of rainfall is 'mm/h'.
- ◯ After obtaining the values of T_L, k and p, Runge-Kutta method is used as a numerical integration method in order to re-product Hydrograph.

4. Examples of analysis

Trial calculation for following cases were done.

- ◯ Case 1 : Mukawa-river (09/Aug/1992)
  (Source: 若手水文学研究会：現場のための水文学 (1) – 流出解析　その 1 – in Japanese character)
- ◯ Case 2 : Sample data from textbook
  (Source: 椎葉充晴・立川康人・市川温，例題で学ぶ水文学，森北出版株式会社，20120 年 5 月 21 日 in Japanese character)
- ◯ Case 3 : Tyuuruigawa-river (05/Aug/1975)
  (Source: 北海道開発局土木試験所河川研究室：実用的な洪水流出計算法，昭和 62 年 3 月 in Japanese character)
- ◯ Case 4 : Otoshibegawa-river (29/Jun/1960)
  (Source: 実用的な洪水流出計算法 (洪水データ集) in Japanese character)
- ◯ Case 5 : Otoshibegawa-river (03/Apr/1961)
  (Source: 実用的な洪水流出計算法 (洪水データ集) in Japanese character)
- ◯ Case 6 : Furumaigawa-river (28/Jun/1966)
  (Source: 実用的な洪水流出計算法 (洪水データ集) in Japanese character)

After this, the relationship between direct run-off and storage, and re-producted Hydrograph using analized parameters are shown for each case.

4.1 Case 1: Mukawa-river (09/August/1992)

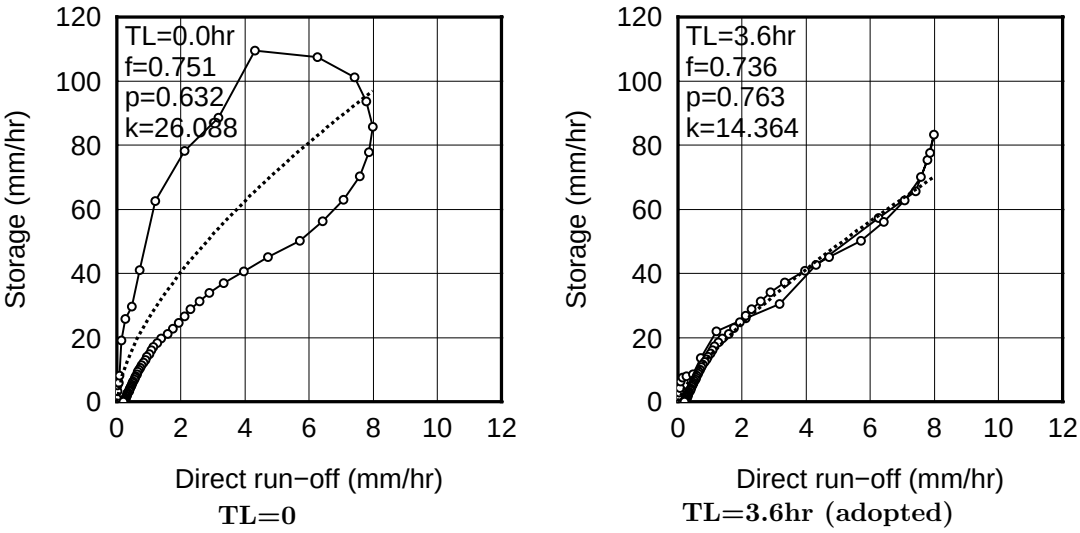
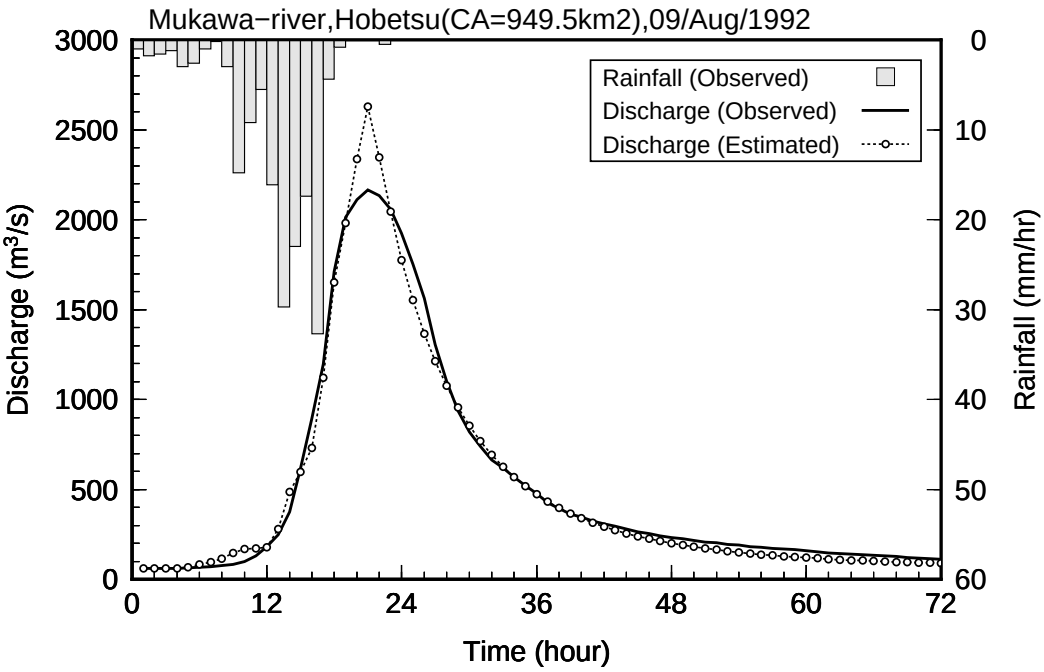


Fig.1 q_c - S **relationship**



n_d	n_1	n_2	Delay time	f	p	k
72	3	72	3.6	0.736	0.763	14.364

Fig.2 Hydrograph for Case 1

4.2 Case 2: Example from textbook

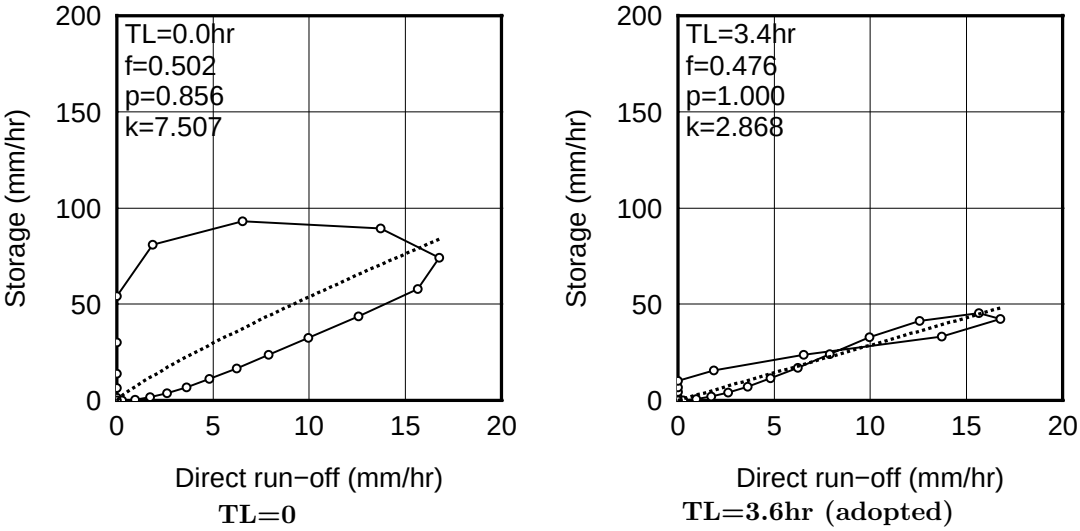
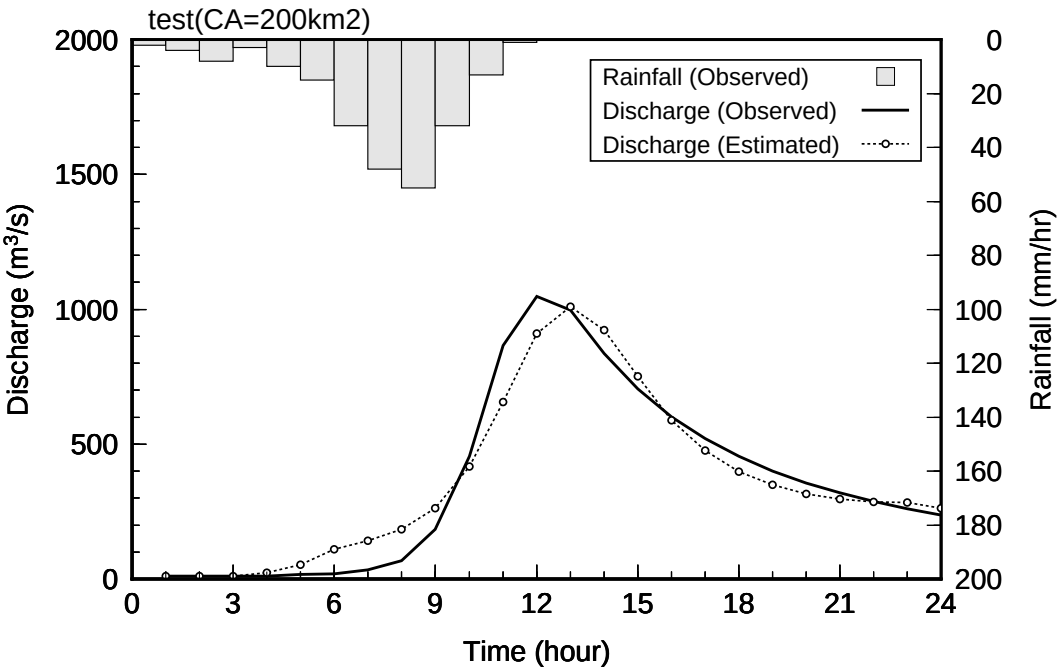


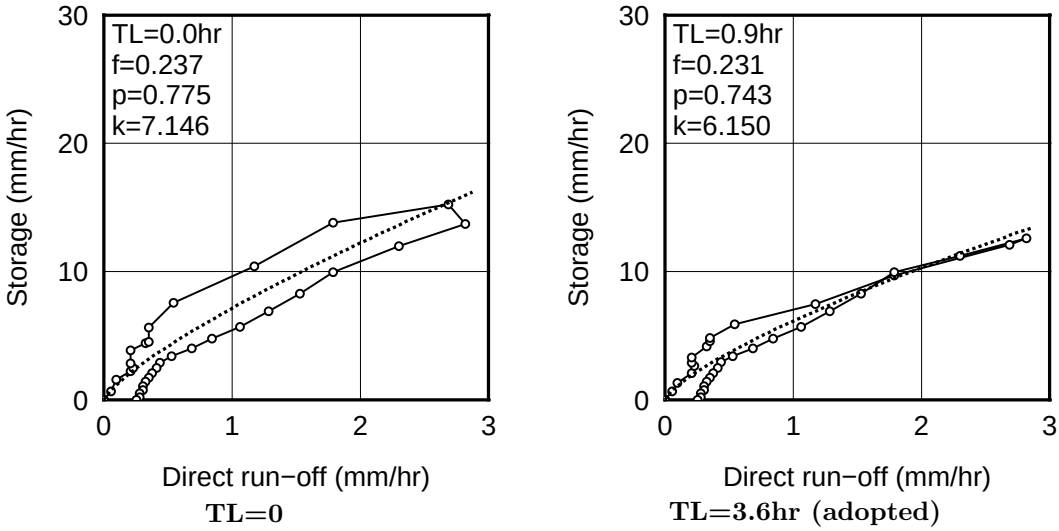
Fig.3 q_c **-** S **relationship**



n_d	n_1	n_2	Delay time	f	p	k
24	3	23	3.4	0.476	1.000	2.868

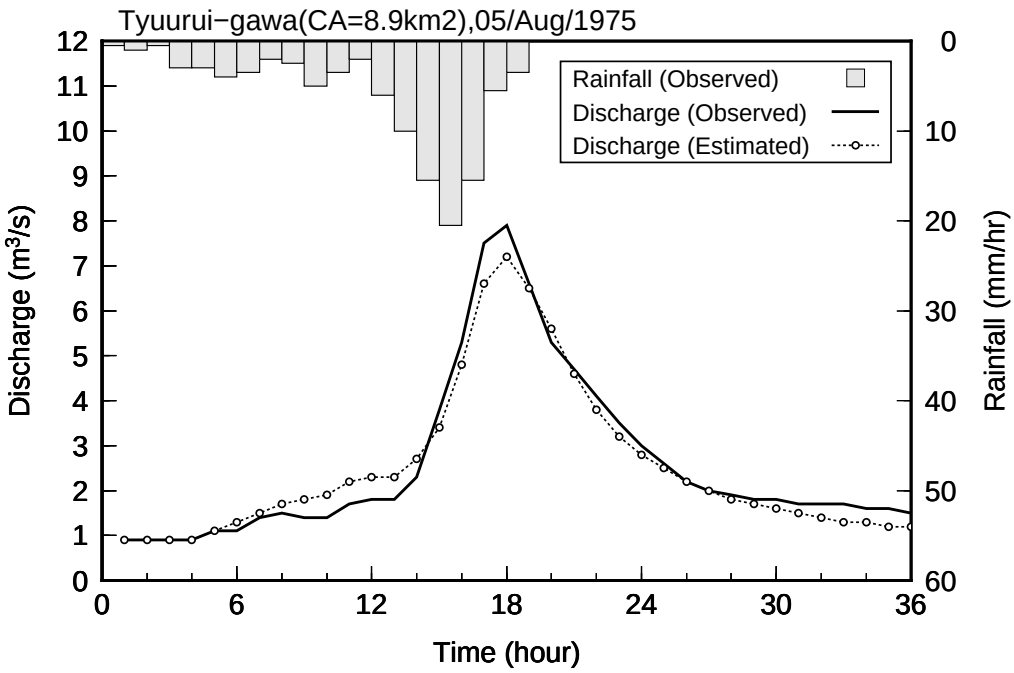
Fig.4 Hydrograph for Case 2

4.3 Case 3: Tyuuruigawa-river (05/August/1975)



TL=0 **TL=3.6hr (adopted)**

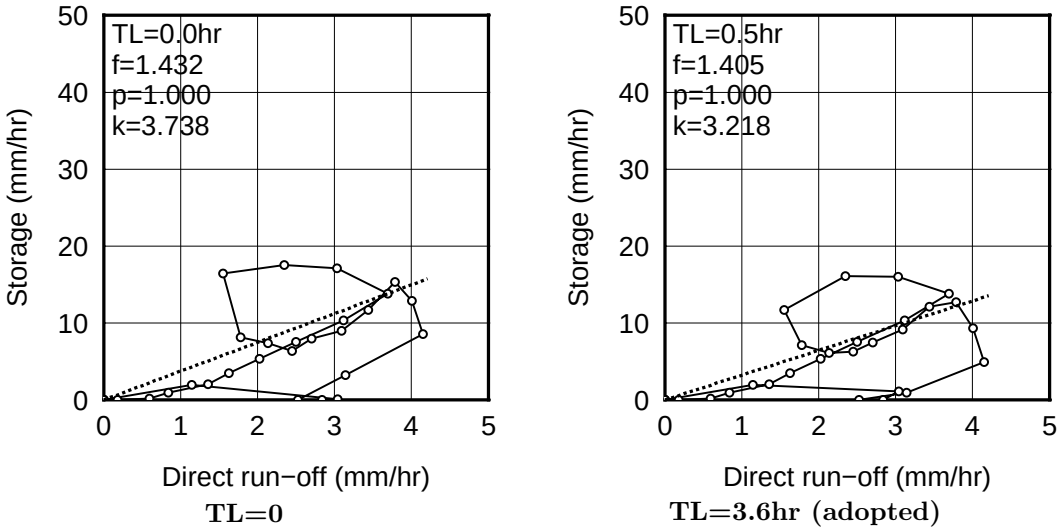
Fig.5 q_c **-** S **relationship**



n_d	n_1	n_2	Delay time	f	p	k
36	4	36	0.9	0.231	0.743	6.150

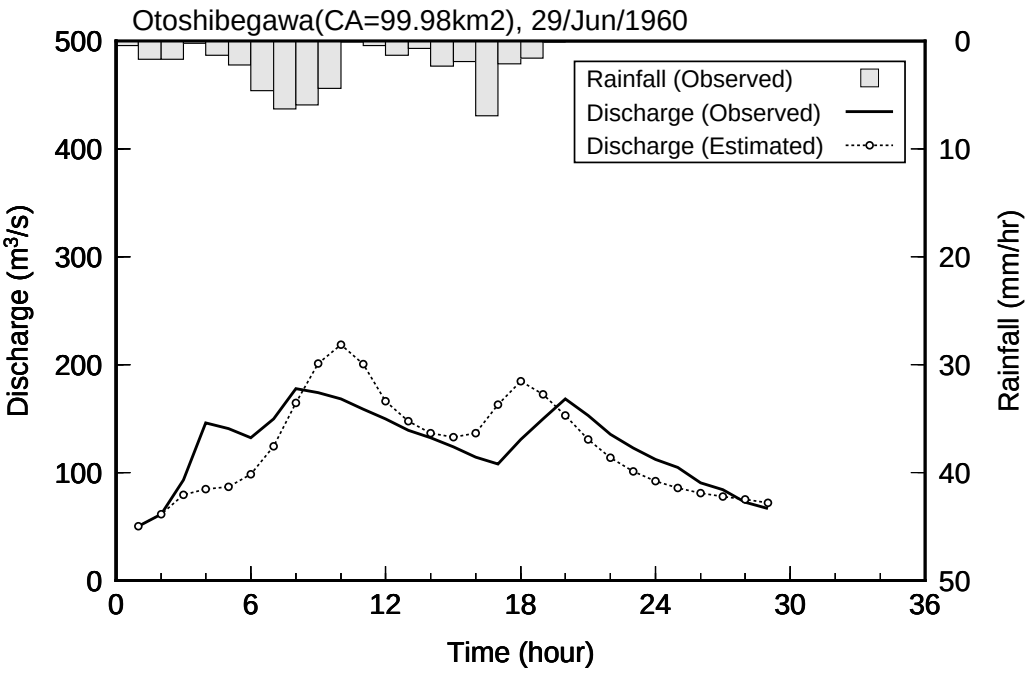
Fig.6 Hydrograph for Case 3

4.4 Case 4: Otoshibegawa-river (29/June/1960)



TL=0 **TL=3.6hr (adopted)**

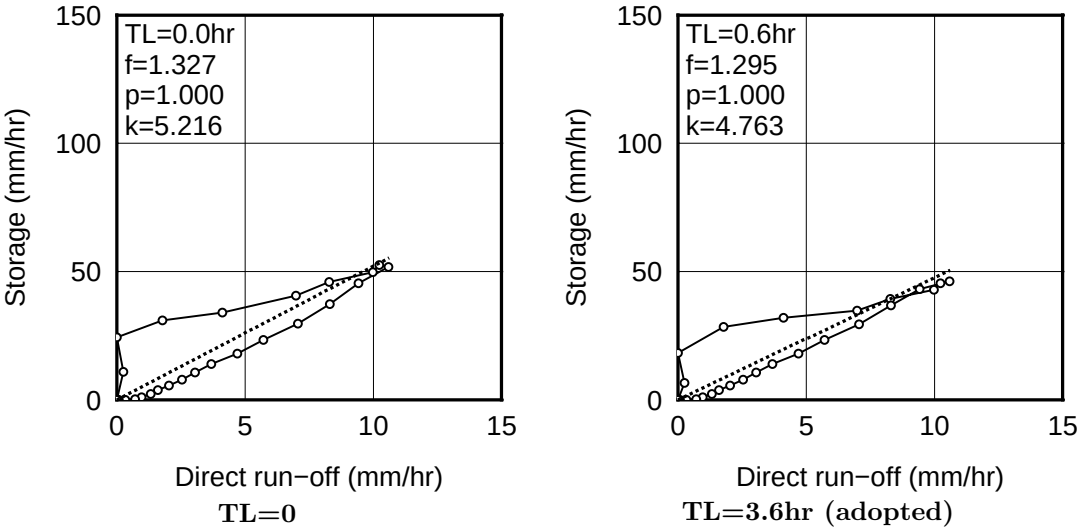
Fig.7 q_c **-** S **relationship**



n_d	n_1	n_2	Delay time	f	p	k
29	2	28	0.5	1.405	1.000	3.218

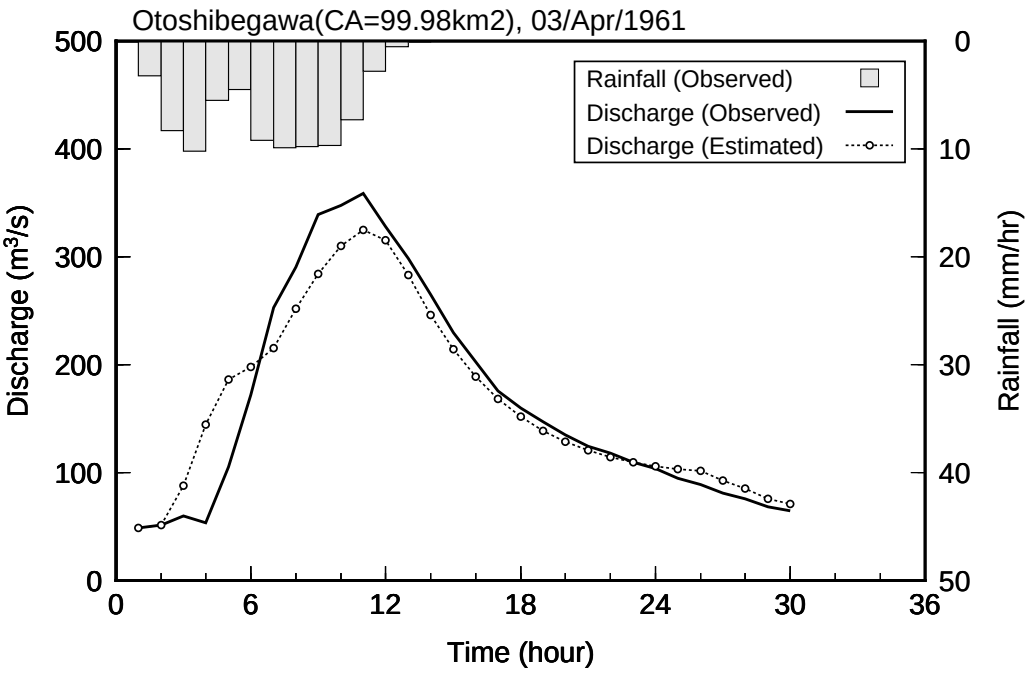
Fig.8 Hydrograph for Case 4

4.5 Case 5: Otoshibegawa-river (03/April/1961)



TL=0 **TL=3.6hr (adopted)**

Fig.9 q_c **-** S **relationship**



n_d	n_1	n_2	Delay time	f	p	k
30	2	26	0.6	1.295	1.000	4.763

Fig.10 Hydrograph for Case 5

4.6 Case 6: Furumaigawa-river (28/June/1966)

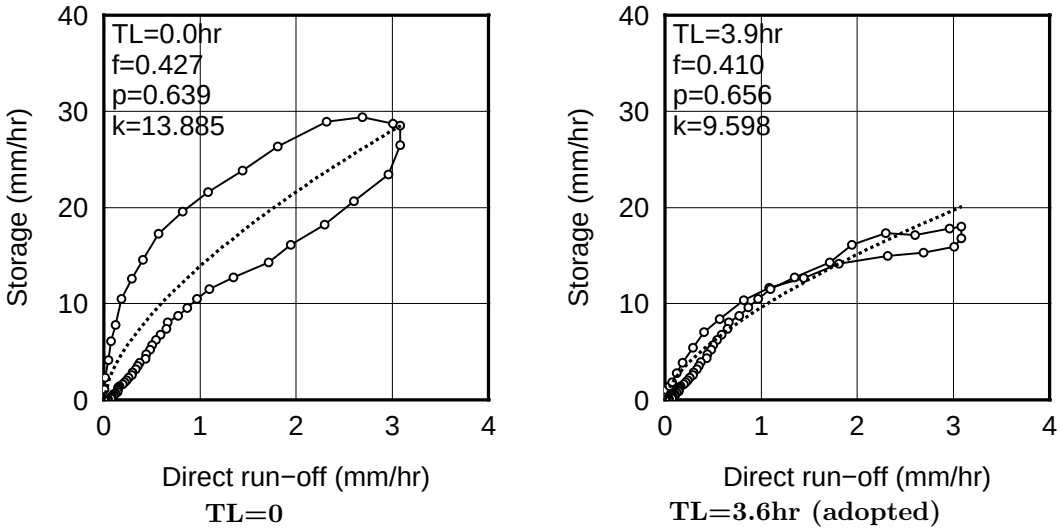
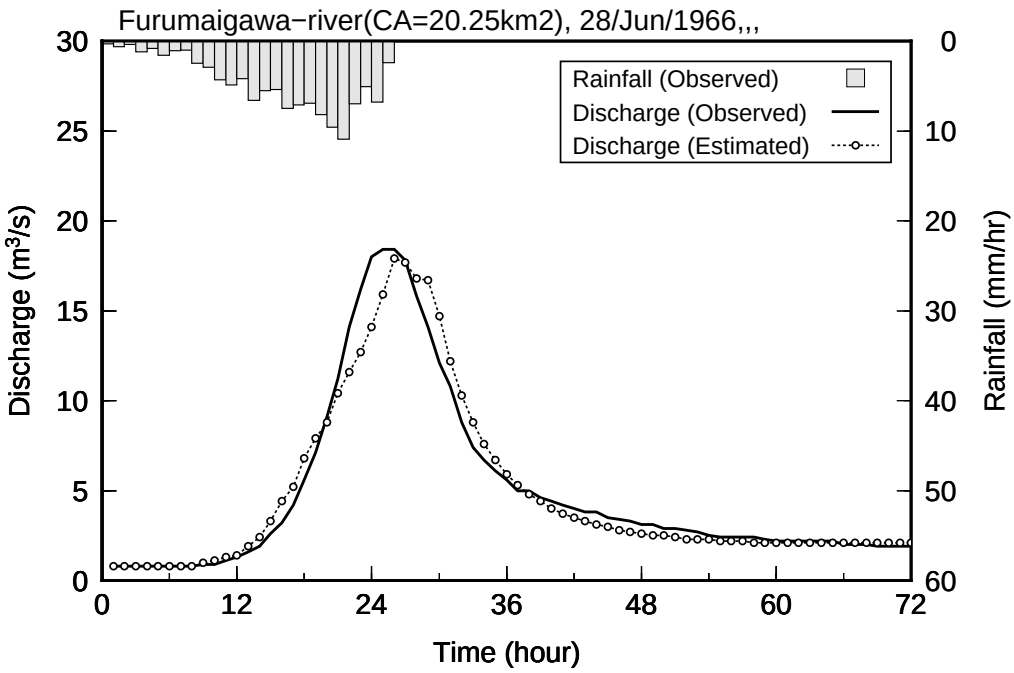


Fig.11 q_c - S **relationship**



n_d	n_1	n_2	Delay time	f	p	k
72	8	72	3.9	0.410	0.656	9.598

Fig.12 Hydrograph for Case 6

References

- ◯ http://www.scj.go.jp/ja/member/iinkai/bunya/doboku/takamizu/pdf/haifusiryou04-1.pdf
  (角屋睦，永井昭博：流出解析法 (その 10) – 4. 貯留法-貯留関数法による洪水流出解析 –，農業土木学会誌第 48 巻第 10 号，pp43-50，1980 年 10 月 in Japanese character)
- ◯ http://kankyou.ceri.go.jp/genba/suimon.html
  (若手水文学研究会：現場のための水文学 (1) – 流出解析　その 1 – in Japanese character)
- ◯ ISBN 978-4-627-49631-6 published 05.2010 (textbook)
  (椎葉充晴・立川康人・市川温，例題で学ぶ水文学，森北出版株式会社，2010 年 5 月 21 日 in Japanese character)
- ◯ http://river.ceri.go.jp/contents/tool/kouzui.html
  (北海道開発局土木試験所河川研究室：実用的な洪水流出計算法, 昭和 62 年 3 月 in Japanese character)
- ◯ http://river.ceri.go.jp/contents/tool/kouzui.html
  (実用的な洪水流出計算法 (洪水データ集) in Japanese character)